6. Incomplete Contracts

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Basic Readings

Textbooks:

- Bolton and Dewatripont (2005), Chapters 11 and 12
- Hart (1995)
- Che (2008)
- Schmidt (1995), Chapter 6 and 7

Papers:

- Grossman and Hart (1986)
- Hart and Moore (1990)
- Nöldeke and Schmidt (1995, 1998)

Introduction

We have to carefully distinguish:

- “complete, state contingent contracts” (Arrow, Debreu)
- “comprehensive contracts” (principal agent theory)
- “incomplete contracts”

Definition 1 (Incomplete Contracts)

“An incomplete contract has gaps, missing provisions, and ambiguities and has to be completed (by renegotiation or by the courts) with strictly positive probability in some states of the world.” (Hart, 1995)

According to this definition most real world contracts are incomplete:

- they are not contingent on all relevant, publicly available information
- they are short-term
- they are renegotiated frequently
- they are interpreted and completed by the courts

Introduction

The theory of incomplete contracts is closely related to the theory of firm:

- If complete contingent contracts could be written, we would not need firms. All transactions that are carried out within firms could be carried out between independent contractors.
- The firm is an attempt to deal with the problems that arise when contracts are incomplete.
- But what exactly is the difference between a transaction carried out within a firm and between two separate firms?
- Which input goods should be produced in-house and which should be contracted out to independent suppliers?
- What determines the degree of vertical and horizontal integration?

To address these questions we start out with a short overview on different theories of the firm.

Neoclassical Theory of the Firm

- A firm is a production function (black box) that transforms inputs into outputs.
- Manager of the firm maximizes profits
- No incentive problems within the firm
- U-shaped average cost function (because some factors of production cannot be redoubled) $\Rightarrow$ boundaries of the firm are determined by the minimum of the average cost curve.

Neoclassical Theory of the Firm

Problems:

- Theory explains the size of a production plant but not the size of the firm.
- "Selective intervention": Why isn't is always optimal to merge two firms? ("Williamson Puzzle")
- The "firm" itself is not well defined. What is the difference between a contract of the owner of the firm with his workers and a contract of the owner with his suppliers? Why do the workers belong to the firm, while the suppliers are outside of the firm? What determines the boundary of the firm?
- Samuelson: In a neoclassical world it does not make a difference whether the capitalists hire workers or whether the workers hire capitalists.

Principal-Agent Theory

Principal-agent theory deals with conflicts of interest and asymmetric information:

- Incentives problems are taken seriously and modeled explicitly.
- Theory characterizes the optimal structure of a “comprehensive” contract.

Problems:

- There is no difference between an incentive contract within a firm and an incentive contract between two seperate firms. Example: General Motors and Fisher Body.
- No solution to the “Williamson Puzzle”.

Conclusion: If comprehensive contracts are being written, the organizational structure does not play a role. Any allocation that can be implemented through a given organizational structure could be implemented within any other organizational structure through the appropriate choice of a comprehensive contract.

Transaction Cost Approach

1. **Coase (1937):** When does a transaction take place within a firm and when is it carried out on a market?
   - Markets: Allocation through the price mechanism
   - Firms: Allocation by fiat

   Different allocation mechanisms give rise to different transaction costs. In small groups fiat is the more efficient allocation mechanism. But: the larger the group (the “firm”), the higher are “bureaucracy costs” and the more efficient is the price mechanism.
2. **Alchian and Demsetz (1972):**
   - No difference between prices and fiat: Grocer story.
   - Theory of property rights based on monitoring. Who monitors the monitor? The owner.

Transaction Cost Approach

3. **Klein, Crawford and Alchian (1978), Williamson (1975, 1985):** Transaction costs in writing a contract induce parties to write incomplete contracts:
   - Costs to think through all possible states of the world.
   - Costs to write down all possible contingencies (using legal code)
   - Difficulty to describe a contingency unambiguously so that it can verified by the courts even if the contingency is obvious to the contracting parties

   Hence, parties write incomplete contracts ex ante that have to be completed and renegotiated as they go along.
   However, this will yield inefficiencies for several reasons:
   - Haggling in renegotiation yields delay and inefficient decisions
   - Asymmetric information may arise during the relationship which prevents the implementation of an ex post efficient allocation.
   - These costs would not matter very much, if the parties could easily switch to alternative contracting parties. However, in many situations the parties are locked in with each other and a break up would be very inefficient. In these situations, the **hold-up problem** arises.

The Hold-Up Problem

The hold-up problem can be described as follows:

- The parties must make “relationship specific investments” ex ante that increase the potential surplus that can be generated in their relationship. These investments are (at least partially) sunk and lose their value when the relationship breaks up.
- It is not possible to contract ex ante on the investments nor on how to share the surplus ex post.
- When the parties negotiate on how to share the surplus ex post, the ex ante investments are already sunk and do not affect the bargaining outcome. Hence, the parties get wrong investment incentives.

The Hold-Up Problem

Examples for relationship specific investments:

- a worker acquires specific skills that are valuable only in one particular firm,
- a worker builds his house close to the firm he works for,
- a company invests in capacity that can only be used for one particular customer,
- a company develops a product that is specific to the needs of one particular customer, etc.

A Formal Hold-Up Model

A buyer (B) and a seller (S) can trade quantity $q \in [0, \overline{q}]$ of a good at some future date 2. At date 1 the seller can make an investment at cost I that reduces his production cost and/or increases the buyer's valuation.

- If the seller invests, his production cost is $c_I(q)$ and the buyer's valuation is $v_I(q)$. In this case the optimal quantity of trade is $q_I = \arg\max\{v_I(q) - c_I(q)\}$ and the gross social surplus is $W_I = v_I(q_I) - c_I(q_I)$.
- If the seller does not invest, his production cost is $c_0(q)$ and the buyer's valuation is $v_0(q)$. In this case the optimal quantity of trade is $q_0 = \arg\max\{v_0(q) - c_0(q)\}$ and the gross social surplus is $W_0 = v_0(q_0) - c_0(q_0)$.

Assume that the investment is efficient, i.e. $W_I - I > W_0$.

A Formal Hold-Up Model

Suppose the buyer and the seller cannot contract on the investment. Furthermore, they can contract on how much to trade only after the investment has been sunk. Nash bargaining results in efficient trade and each party getting half of the gross social surplus:

- If the seller invested the buyer gets $U_B(I) = \frac{1}{2}W_I$ and the seller gets $U_S(I) = \frac{1}{2}W_I - I$.
- If the seller did not invest the buyer gets $U_B(0) = \frac{1}{2}W_0$ and the seller gets $U_S(0) = \frac{1}{2}W_0$.

Thus, the seller will not invest if

$$\frac{1}{2}W_I - I < \frac{1}{2}W_0$$

even if the investment is efficient. The reason is that he has to bear the full cost of investment but gets only half of the benefits.

A Formal Hold-Up Model

Implicit Assumptions of this Model:

1. Investment cannot be contracted upon. Otherwise S and B could write a contract to share the cost of the investment such that the investment pays off for S.
2. A long-term contract governing the ex post division of the surplus is not feasible either. Otherwise the parties could give S a sufficiently large share of the gross social surplus that he is willing to invest.
3. A long-term contract governing the quantity of trade is not feasible either. Otherwise the parties could use such a contract to implement the investment (see below).

A Formal Hold-Up Model

Vertical Integration

Williamson (1979) and Klein et.al. (1978) argue that the hold-up problem can be solved by vertical integration: If B buys S, then he can force S to invest and to produce the efficient quantity q_I.

Problems:

- This argument explains the benefits of integration only. What is the downside?
- It is not clear why S's bargaining power is so much reduced once he has been bought by B.

These problems are solved by the property rights approach of Grossman, Hart and Moore.

The Property Rights Approach

The early "Property Rights" literature emphasized the importance of clearly defined property rights (Coase, 1960). But, this theory cannot explain to whom the property rights of an asset should be allocated.

Grossman and Hart (1986)

Definition 2 (Ownership Rights)

"An owner of an asset has the residual control rights over that asset: the right to decide all usages of the asset in any way not inconsistent with a prior contract, custom, or law." (Hart, 1995, p.30)

This definition is quite close to the definition of ownership in German law:

> *BGB, §903 [Befugnisse des Eigentümers:* [1] *Der Eigentümer einer Sache kann, soweit nicht das Gesetz oder Rechte Dritter entgegenstehen, mit der Sache nach Belieben verfahren und andere von jeder Einwirkung ausschließen.*

The Property Rights Approach

Remarks:

- “Specific control rights” can be contracted away.
- “Residual control rights” remain with the owner.
- Distinction between “physical capital” and “human capital”. Ownership on physical assets can be traded but not ownership on human assets.
- Definition of a **“firm”:** “A firm consists of the physical assets that it owns.”
- Grocer story: A owns the only restaurant, B the only grocery in town. A may supply a large convention for which he needs the groceries of the grocery store.
  - Separate ownership: A and B bargain on how to share the profits that can be made from the convention and share the surplus 50:50. If A wants to fire B, he has to fire B and his grocery.
  - Joint ownership: If A bought B’s grocery, he can “order” B to deliver the groceries. If B refuses to deliver, A can fire him and employ somebody else, because he continues to own the physical capital (the grocery). Thus, he will get the entire surplus.

The Property Rights Approach

- Control of physical capital can lead to control over human capital!
- The allocation of ownership rights affects the bargaining power of the two parties and thus how they share the surplus, which may in turn affect their ex ante investment incentives.

The Grossman-Hart Model

Consider again the buyer-seller hold-up model that we introduced before. We extend the model by introducing two assets, A_1 and A_2 that are required for the realization of the social surplus.
Asset ownership affects the outside option utilities of the two parties if the negotiations at date 2 fail. Of course, in equilibrium negotiations never fail and the parties always achieve ex post efficiency. However, the outside option utilities affect the bargaining power of the two parties. Thus, allocating asset ownership affects the bargaining power at the negotiation stage and thereby the share of the surplus that the seller gets.

There are three possible ownership structures:

- BI (buyer integration): The buyer owns both assets
- SI (seller integration): The seller owns both assets
- NI (non-integration): The seller owns A_1 and the buyer owns A_2.

The Grossman-Hart Model

Under i-integration, $i \in \{B, N, S\}$, party j's outside option utility is given by $\underline{U}_j^i$. Grossman and Hart impose the following assumptions:

Assumption 6.1

(i) $\underline{U}_B^i(I) + \underline{U}_S^i(I) < W_I$ *and* $\underline{U}_B^i(0) + \underline{U}_S^i(0) < W_0$ *for all* $i \in \{B, N, S\}$

(ii) $\underline{U}_j^i(I) - \underline{U}_j^i(0) < W_I - W_0$ *for all* $i \in \{B, N, S\}$ *and* $j \in \{B, S\}$

(iii) $\underline{U}_S^S(I) - \underline{U}_S^S(0) > \underline{U}_S^N(I) - \underline{U}_S^N(0) > \underline{U}_S^B(I) - \underline{U}_S^B(0) = 0$ *and*
$\underline{U}_B^B(I) - \underline{U}_B^B(0) > \underline{U}_B^N(I) - \underline{U}_B^N(0) > \underline{U}_B^S(I) - \underline{U}_B^S(0) = 0$

Interpretation:

1. Assumption (i) says that trade in the relationship is efficient: it is always better for the two parties to trade with each other rather than to take their outside options.
2. Assumption (ii) says that the investment is relationship specific: It increases the social surplus within the relationship more than it increases the outside option utilities.

3. Assumption (iii) says that the investment increases the outside option of a party only if this party has access to the assets. If a party does not own any asset then it's outside option is unaffected by the investment.

Consider now the negotiation at date 2. If the seller invested at date 1 his payoff under i-integration is

$$\begin{aligned} U_S^i(I) &= \underline{U}_S^i(I) + \frac{1}{2}\left[W_I - \underline{U}_S^i(I) - \underline{U}_B^i(I)\right] - I \\ &= \frac{1}{2}W_I + \frac{1}{2}\left[\underline{U}_S^i(I) - \underline{U}_B^i(I)\right] - I \end{aligned}$$

Similar, if the seller did not invest at date 1 his payoff under i-integration is

$$\begin{aligned} U_S^i(0) &= \underline{U}_S^i(0) + \frac{1}{2}\left[W_0 - \underline{U}_S^i(0) - \underline{U}_B^i(0)\right] \\ &= \frac{1}{2}W_0 + \frac{1}{2}\left[\underline{U}_S^i(0) - \underline{U}_B^i(0)\right] \end{aligned}$$

The Grossman-Hart Model

Thus, the seller will invest iff

$$\frac{1}{2}W_I + \frac{1}{2}\left[\underline{U}_S^i(I) - \underline{U}_B^i(I)\right] - I > \frac{1}{2}W_0 + \frac{1}{2}\left[\underline{U}_S^i(0) - \underline{U}_B^i(0)\right]$$

which is equivalent to

$$\frac{1}{2}[W_I - W_0] + \frac{1}{2}\left[\underline{U}_S^i(I) - \underline{U}_S^i(0)\right] - \frac{1}{2}\left[\underline{U}_B^i(I) - \underline{U}_B^i(0)\right] > I$$

Note that given Assumption 6.1 (iii) the left hand side is largest under SI, second largest under NI and smallest under BI. Thus, the seller's incentives to invest are maximized when he owns all the assets.

Intuition:

1. Only half of the investment incentives are given by the investment's effect on social surplus. The other half stems from the effect of the investment on the seller's outside option.
2. Seller integration maximizes the marginal impact of the investment on the seller's outside option.
3. Therefore the seller should own all the assets.

The Grossman-Hart Model

Conclusions:

- Asset ownership affects the outside option utilities of the parties and thereby the share of the surplus they get in the bargaining game.
- In our model only one party had to invest and the investment decision was binary.
- Grossman and Hart (1986) consider a more general model in which both parties have to make an investment and where the investments are continuous.
- Grossman and Hart show that typically no ownership structure implements the first best. However not all ownership structures are equally inefficient. There is a second-best allocation of ownership rights.
- If the investment of one party is particularly important, then it is often optimal to give this party all the ownership rights.
- If both investments are important it may also be optimal to have non-integration.

The Grossman-Hart Model

- Joint ownership is never optimal because it minimizes the outside option utilities and thus the investment incentives.
- The theory explains the benefits and costs of integration simultaneously. The benefit of integration is that it increases the investment incentives of the owner of the integrated firm. The cost is that it reduces investment incentives of the non-owner. Thus, the model provides an answer to Coase's question why it is not optimal to organize all production in one big firm. It also shows why Williamson's idea of selective intervention does not always work. After the firm is integrated under B-ownership it is impossible to replicate the investment incentives that the seller had under non-integration.

The Hart-Moore (1990) Model

Description of the Model:

- $\underline{S} = \{1, \ldots, I\}$ set of agents
- $\underline{A} = \{a_1, \ldots, a_n\}$ set of physical assets
- $x_i \in [0, \overline{x}_i]$ investment of agent i at date 1.
- $C_i(x_i)$ cost of x_i
- $v(S, A \mid x)$ surplus that can be generated by coalition $S \subseteq \underline{S}$ if it controls the set of assets $A \subseteq \underline{A}$ and if the investments $x = (x_1, \ldots, x_I)$ have been made.

Assumption 6.2 (Technological Assumptions)

A1 $C_i(x_i) \geq 0$ *is strictly increasing, differentiable, and strictly convex with* $\lim_{x_i \to 0} C_i'(x_i) = 0$ *and* $\lim_{x_i \to \bar{x}_i} C_i'(x_i) = \infty$ *(Inada conditions).*

A2 $v(S, A \mid x) \geq 0$ *is differentiable and concave in* x

A3 $\frac{\partial v(S,A,|x)}{\partial x_i} \equiv v^i(S, A, \mid x) = 0$ *if* $i \notin S$. *(Investment is in human capital, because it increases the value of a coalition only if* i *is a member of the coalition).*

A4 $\frac{\partial v^i(S,A,|x)}{\partial x_j} \geq 0 \;\; \forall j \neq i$ *(investments are complements).*

A5 *For all* $S' \subseteq S$ *and* $A' \subseteq A$ *we have:*

$$v(S, A, \mid x) \geq v(S', A' \mid x) + v(S \backslash S', A \backslash A' \mid x)$$

("superadditivity of total surplus" $\Rightarrow$ *total surplus is maximized in the grand coalition.*

A6 *For all* $S' \subseteq S$ *and* $A' \subseteq A$ *we have:* $v^i(S, A, \mid x) \geq v^i(S', A' \mid x)$ *("super additivity of marginal surplus").*

Control Structures:

A control structure $\alpha(S)$ describes for any coalition $S \subseteq \underline{S}$, the function $\alpha(S)$ tells us, which assets this coalition controls. Examples:

- Agent i owns asset a_n: $a_n \in \alpha(S) \Leftrightarrow i \in S$
- Agent i has veto power on a_n: $a_n \in \alpha(S) \Rightarrow i \in S$
- Agent i owns share $\sigma_n(i)$ of a_n: $a_n \in \alpha(S) \Leftrightarrow \sum_{i \in S} \sigma_n(i) > 0.5$.

The Hart-Moore (1990) Model

Assumption 6.3 (Contractual Assumptions)

C1 *x_i is observable, but not verifiable (investments are non contractible).*

C2 *No ex ante contracts on the sharing of the surplus possible. Parties have to negotiate on how to share the surplus ex post.*

C3 *Ex ante (at date 0) only contracts on the control structure $\alpha(S)$ are feasible.*

C4 *Bargaining at date 2 always yields an ex post efficient outcome that maximizes total surplus which is shared according to the Shapley value:*

$$B_i(\alpha \mid x) \;=\; \sum_{S \mid i \in S} p(S)\left[v(S, \alpha(S) \mid x) - v(S \backslash \{i\}, \alpha(S \backslash \{i\}) \mid x)\right]$$

where $p(S) \;=\; \frac{(|S|-1)!(I-|S|)!}{I!}$.

The Hart-Moore (1990) Model

Digression: The Shapley value is a generalization of the Nash Bargaining Solution to N player cooperative games with transferable utility. The Shapley value is the only solution concept that satisfies the following axioms:

1. **Symmetry:** If two players are perfect substitutes for each other, they should get the same payoff.
2. **Dummy Player:** If a player does not contribute to any possible coalition, then his payoff is 0.
3. **Efficiency:** The sum of all payoffs equals the value of the grand coalition.
4. **Additivity:** The value of the sum of two games is equal to the sum of the values of these two games.

The Shapley value gives each player his “expected” marginal contribution to all possible coalitions.

Many applications of the Shapley value in market games, in large exchange economies, in accounting, in political science, etc.

The First Best Allocation

$$\max_x v(\underline{S}, \underline{A} \mid x) - \sum_{i=1}^{I} C_i(x_i)$$

FOC:

$$v^i(\underline{S}, \underline{A} \mid x^{FB}) \;=\; C_i'(x_i^{FB})$$

By A1 and A2, FOC is a necessary and sufficient condition for an optimal solution.

Proposition 6.1 (Underinvestment)

Any control structure α implements (weak) underinvestment in the Nash equilibrium of the investment game at stage 1:

$$x_i^*(\alpha) \leq x_i^{FB}$$

The First Best Allocation

Proof sketch: Each agent maximizes $B_i(\alpha \mid x) - C_i(x_i)$. In NE we must have:

$$\frac{\partial}{\partial x_i} B_i(\alpha \mid x^*) = \sum_{S|i \in S} p(S) v^i(S, \alpha(S) \mid x^*(\alpha)) = C_i'(x_i)$$

Note that the $p(S)$ add up to 1. But $v^i(S, \alpha(S)) \leq v^i(\underline{S}, \underline{A})$ by A6.
Suppose all other agents would choose $x_j^* = x_j^{FB}$, $j \neq i$. Then player i would have an incentive to invest too little. But this (weakly) reduces the investment incentives of the other players $\Rightarrow$ (weak) underinvestments. *Q.E.D.*

Remarks:

1. Note that investments are strategic complements here. A more elegant proof would use the properties of supermodularity.
2. Without A6 overinvestments would also be possible.

The Yacht Example, Variant 1

- One asset: luxury yacht in the Caribbean.
- Three agents: skipper (1), chef (2), tycoon (3).
- Date 2: 1 and 2 prepare a luxury dinner for 3.
- Date 1: Chef can invest(create a special dish), the investment costs $C_2 = 100$ and increases tycoons valuation by 240.
- Investment is **asset specific** (there is only one Yacht at this island)
- Skipper and chef are **dispensable** (there are many unemployed skippers and chefs on this island).
- Tycoon is **indispensable** (there is only one tycoon on the island who can afford this dinner).

Who should own the yacht?

Case 1: The skipper owns the yacht.

How will the gross surplus be distributed if the chef invested?

possible permutations			marginal contribution of Skipper	Chef	Tycoon
S	C	T			
S	T	C			
C	S	T			
C	T	S			
T	S	C			
T	C	S			
	$\sum$				
	$\frac{1}{6}\sum$				

Why is the chef not going to invest if the skipper owns the yacht?

The Yacht Example 1

Case 2: The chef owns the yacht.

possible permutations			marginal contribution of Skipper	Chef	Tycoon
S	C	T			
S	T	C			
C	S	T			
C	T	S			
T	S	C			
T	C	S			
	$\sum$				
	$\frac{1}{6}\sum$				

Why is the chef going to invest if he owns the yacht?

Case 3: The tycoon owns the yacht

This case is equivalent to case 2: Only the chef and the tycoon are required to generate the surplus.

The Yacht Example 1

Proposition 6.2

If only one agent has to take an investment, then he should own all the assets.

Proof sketch: We want to maximize i's investment incentives (why?):

$$\frac{\partial B_i(\alpha)}{\partial x_i} = \sum_{S|i \in S} p(S) v^i(S, \alpha(S))$$

By A6 this term is maximized if $\alpha(S) = \underline{A}$ for all $S \mid i \in S$. *Q.E.D.*

Definition: An agent i is **indispensable** for an asset a_n if a_n does not affect the marginal product of any investment if i is not a member of the coalition, i.e.,

$$v^j(S, A) = v^j(S, A \backslash \{a_n\}) \text{ if } i \notin S$$

Proposition 6.3

If an agent is indispensable for an asset, he should own it.

The Yacht Example 1

Proof sketch: Suppose i does not own a_n in the original control structure. Now we transfer the ownership rights on a_n to i and leave all other ownership rights unaltered.

- i's investment incentives improve, because now there are some additional coalitions to which i belongs that control a_n and did not do so before.
- The investment incentives of all other players cannot be reduced. In the original control structure there were some coalitions to which i belonged that did not control a_n. At the same time there were some coalitions controlling a_n to which i did not belong. In all of these coalitions a_n was useless and did not affect anybody's investment incentives. Now there are some coalitions that neither contain a_n nor i (here investment incentives don't change) and some coalitions, that contain both i and a_n (here investment incentives improve).

Q.E.D.

The Yacht Example, Variant 2

- Variant 1, but in addition:
- The skipper can invest, too, e.g. by learning the history of the local islands
- $C_1 = 100$, increased valuation of the tycoon is 240.

Who should own the yacht?

The Yacht Example 2

Case 1: The tycoon owns the yacht.

How will the gross surplus be distributed if the chef and the skipper invested?

possible permutations			marginal contribution of Skipper	Chef	Tycoon
S	C	T			
S	T	C			
C	S	T			
C	T	S			
T	S	C			
T	C	S			
$\sum$					
$\frac{1}{6}\sum$					

Why are the chef and the skipper investing, even though the tycoon owns the yacht?

The Yacht Example 2

Case 2: The chef owns the yacht.

possible permutations			marginal contribution of Skipper	Chef	Tycoon
S	C	T			
S	T	C			
C	S	T			
C	T	S			
T	S	C			
T	C	S			
$\sum$					
$\frac{1}{6}\sum$					

Who is going to invest in this case?

Case 3: The skipper owns the yacht

Who is going to invest in this case?

The Yacht Example, Variant 3

- Like variant 2, but in addition
- Yacht consists of two parts, the galley and the hull, which are complementary.
- the tycoon is dispensable (many potential tycoons)
- tycoon can also make an investment (increased valuation of the tycoon 240).
- investment cost: C_i, $i \in \{1, 2, 3\}$

Compare two ownership structures:

1. Non-integration: chef owns the galley, skipper owns the hull.
2. Integration: Chef owns galley and hull.

Which ownership structure is more efficient?

The Yacht Example 3

	Chef invests if and only if	Skipper invests if and only if	Tycoon invests if and only if
Non-Integration	$C_1 \leq 120$	$C_2 \leq 120$	$C_3 \leq 80$
Integration	$C_1 \leq 240$	$C_2 \leq 120$	$C_3 \leq 120$

Intuition:

- Non-integration: The tycoon requires the chef and the skipper to generate his surplus of 240, so he gets only 80. The skipper and the chef only require each other to generate their surplusses of 240, so they get 120.
- Integration: The tycoon only requires the chef to generate his surplus of 240, so he gets 120. The same holds for the skipper. The chef can generate his surplus alone, so he gets 240.

The Yacht Example 3

Definition 3 (Complementary Assets)

Two assets a_n and a_m are **complementary** if they are unproductive if they cannot be used together, i.e.:

$$v^i(S, A) \geq v^i(S, A \backslash \{a_m\}) = v^i(S, A \backslash \{a_n\}) = v^i(S, A \backslash \{a_m, a_n\})$$

$\forall\, i$, A and S, $i \in S$.

Proposition 6.4

If two or more assets are complementary, they should be owned together.

Proof sketch: Suppose a_n and a_m are controlled separately under control structure α. Consider the new control structure $\hat{\alpha}$, that coincides with α except that $a_n \in \hat{\alpha}(S) \Leftrightarrow a_m \in \hat{\alpha}(S)$.

The Yacht Example 3

- In all coalitions that controlled a_n (a_m) but not a_m (a_n) under $\alpha(S)$ and that do not control any of the two assets under $\hat{\alpha}(S)$, investment incentives remain unchanged.
- In those coalition that controlled a_n (a_m) but not a_m (a_n) under $\alpha(S)$ and that do now control both of them, investment incentives are improved.
- All other coalitions are unaffected.

Q.E.D.

If you read the Hart-Moore paper you will find several other interesting results:

- At most one agent should have veto over an asset (no “supermajority rules”, no unanimity rules)
- If two coalitions $S_1, \ S_2$ and two sets of assets $A_1, \ A_2$ are **economically independent** of each other, then S_1 should own A_1 and S_2 should own A_2.
- etc.

Contractual Solutions to the Hold-up Problem

Grossman, Hart and Moore assume that the only contracts that can be written at date 0 (before the investments have to be made) are contracts on ownership rights. However, at date 2 contracts on the quantity of trade can be written.

> ***Why can't we write a contract on the quantity of trade at date 0 already?***

GHM argue that at date 0 the parties do not know which state of the world is going to materialize. There are many possible states of the world that are impossible to describe in a contract at date 0. However, at date 2 the state of the world has materialized. Thus, it is now easy to write a contract on trade because it does not have to be conditional on the realization of the state of the world.

Contractual Solutions to the Hold-up Problem

There has been a big controversy about the validity of this argument.

1. **Hart and Moore (1988)** propose a model in which parties can write a contract on trade at date 1 already, but there is still inefficient investment. They impose two important assumptions:
   - The parties cannot commit not to renegotiate the initial contract, and the bargaining power in the renegotiation game is fixed and exogenously given.
   - If there is no trade at date 2 the courts cannot figure out whether the seller did not deliver or the buyer refused to take delivery.
2. **Aghion, Dewatripont and Rey (1994)** and **Chung (1991)** show that the first best can be achieved if the parties can allocate the bargaining power at the renegotiation game and give all the bargaining power to one of the parties.
3. **Nöldeke and Schmidt (1995)** show that the first best can be achieved by giving the seller the option to trade some quantity at some price, both of which are specified in the date 1 contract. The option plus renegotiation implement the first best.

Contractual Solutions to the Hold-up Problem

4. **Edlin and Reichelstein (1996)** show that a specific performance contract (forcing the seller to produce some quantity at some price, specified in the date 1 contract) can implement the first best if the buyer has all the bargaining power.
5. **Maskin and Tirole (1999)** assume that the parties cannot describe the possible contingencies at date 1. However, they know the payoff consequences of these contingencies. Maskin and Tirole show this is already enough to construct a mechanism that induces the parties to invest efficiently. However, this result requires that the mechanism cannot be renegotiated.
6. **Che and Hausch (1999)** consider the case where investments are cooperative (the investment of seller increases the buyer's valuation and the investment of the buyer reduces the seller's cost). In this case the best contract that can be written at date 1 is the null contract.

Contractual Solutions to the Hold-up Problem

7. **Segal (1999)** and **Hart and Moore (1999)** show that the first best cannot be achieved if there are many potential goods to be traded at date 1 but only one of these goods becomes relevant while all others yield a surplus of 0. Their results require that the parties cannot commit not to renegotiate.

In the following I will use our simple hold-up model with one-sided investment to show how the first best may or may not be implemented if a contract on q is possible at date 0.
Suppose the parties write a contract at date 0 saying that the seller must deliver $\hat{q}$ to the buyer and that the buyer must pay the seller $\hat{p}$ for this.

Of course, if this contract turns out to be inefficient, the parties will renegotiate it at date 2.

Contractual Solutions to the Hold-up Problem

What is the payoff of the seller if he invested and if he did not invest:

$$
\begin{aligned}
U_S(I) &= \hat{p} - c_I(\hat{q}) + \frac{1}{2}\left[W_I - (\hat{p} - c_I(\hat{q}) + v_I(\hat{q}) - \hat{p})\right] - I \\
&= \hat{p} - c_I(\hat{q}) + \frac{1}{2}\left[W_I - (v_I(\hat{q}) - c_I(\hat{q}))\right] - I \\
U_S(0) &= \hat{p} - c_0(\hat{q}) + \frac{1}{2}\left[W_0 - (\hat{p} - c_0(\hat{q}) + v_0(\hat{q}) - \hat{p})\right] \\
&= \hat{p} - c_0(\hat{q}) + \frac{1}{2}\left[W_0 - (v_0(\hat{q}) - c_0(\hat{q}))\right]
\end{aligned}
$$

Thus, the seller is going to invest iff

$$
\begin{aligned}
& U_S(I) - U_S(0) \\
&= c_0(\hat{q}) - c_I(\hat{q}) + \frac{1}{2}\left[W_I - W_0 + c_I(\hat{q}) - c_0(\hat{q}) + v_0(\hat{q}) - v_I(\hat{q})\right] - I \\
&= \frac{1}{2}\left[c_0(\hat{q}) - c_I(\hat{q})\right] + \frac{1}{2}\left[W_I - W_0\right] + \frac{1}{2}\left[v_0(\hat{q}) - v_I(\hat{q})\right] - I \geq 0
\end{aligned}
$$

Contractual Solutions to the Hold-up Problem

Is it possible to find a $\hat{q}$ that induces the seller to invest? This depends on the nature of the investments.

Case 1: Selfish Investment

Suppose that the investment of the seller only reduces his own costs but leaves the valuation of the buyer unaffected. In this case

$$v_0(\hat{q}) - v_I(\hat{q}) = 0$$

. Thus we can write:

$$\begin{aligned} U_S(I) - U_S(0) &= \frac{1}{2}[c_0(\hat{q}) - c_I(\hat{q})] + \frac{1}{2}[W_I - W_0] - I \\ &= \frac{1}{2}[(v_I(\hat{q}) - c_I(\hat{q})) - (v_0(\hat{q}) - c_0(\hat{q}))] + \frac{1}{2}[W_I - W_0] - I \end{aligned}$$

Suppose we set $\hat{q} = q_I^*$. Then we have

- $v_I(q_I^*) - c_I(q_I^*) = W_I$ (by definition)
- $v_0(q_I^*) - c_0(q_I^*) \leq W_0$ (by revealed preference)
- Thus, $U_S(I) - U_S(0) \geq W_I - W_0 - I$.

Contractual Solutions to the Hold-up Problem

Suppose now that we set $\hat{q} = q_0^*$. Then we have

- $v_I(q_0^*) - c_I(q_0^*) \leq W_I$ (by revealed preference)
- $v_0(q_0^*) - c_0(q_0^*) = W_0$ (by definition)
- Thus, $U_S(I) - U_S(0) \leq W_I - W_0 - I$.

Thus, by continuity there exists a $\hat{q}$, $q_0^* \leq \hat{q} \leq q_I^*$, such that $U_S(I) - U_S(0) = W_I - W_0 - I$. Thus, the seller gets exactly the full social return of his investment and has an incentive to invest efficiently.

Case 2: Cooperative Investment

Suppose now that the investment of the seller increases the valuation of the buyer but leaves the cost of the seller unaffected. In this case

$$c_I(\hat{q}) - c_0(\hat{q}) = 0$$

.
Thus we can write:

$$\begin{aligned} U_S(I) - U_S(0) &= \frac{1}{2}[W_I - W_0]] - \frac{1}{2}[v_I(\hat{q}) - v_0(\hat{q})] - I \\ &\leq \frac{1}{2}[W_I - W_0] - I \end{aligned}$$

Note that $v_I(q) > v_0(q)$ for all $q > 0$ and $v_I(q) = v_0(q)$ for $q = 0$. Thus, the best contract is the null contract, specifying $\hat{q} = 0$, but this is equivalent to not writing any contract at all.

Contractual Solutions to the Hold-up Problem

Note:

- If the investment is selfish, we can make the investment more profitable by giving the seller the option to sell a large quantity. This improves his threatpoint utility, or better, the marginal investment incentives stemming from his outside option.
- If the investment is cooperative, there is no benefit to the seller if he sells a lot. To the contrary, the more he has to sell, the worse his is threatpoint utility, or better, the marginal investment incentives stemming from his outside option. Therefore, the best outside option is that he does not have to trade at all.

Options on Ownership Rights

Let us go back to GHM and assume that at date 0 only contracts on the allocation of ownership rights can be written. GHM consider only unconditional ownership structures:

- Integration: One party owns all the assets
- Non-Integration: One party owns some assets, an other party owns some other assets
- Joint Ownership: All assets are jointly owned by both parties
- Majority Voting: Each party owns a share of the assets. Control over the assets is exercised by majority vote.
- etc.

Options on Ownership Rights

Nöldeke and Schmidt (1998) point out that there are also slightly more sophisticated ownership structures.

> ***Example:*** *A is the sole owner of the assets, but B has the option to buy the assets at a fixed price at some future date.*

Compared to GHM this **conditional ownership structure** is somewhat more complicated but it does not require that any additional variables are observable or verifiable.

In reality conditional ownership structures are frequently observed:

- stock options
- convertible securities
- venture capital financing
- Mannesmann-Arcor

In the following I will show how “options on ownership rights” can be used to implement the first best.

The Nöldeke-Schmidt (1998) Model

Introduction

- two parties: A, B
- relationship specific investments in phsysical and/or human capital: $a,\ b\ \in\ \mathbb{R}_0^+$. Investments are sequential.
- physical asset K
- potential surplus: $v(a,b)$ is differentiable, monotonically increasing, strictly concave, and satisfies

$$\lim_{a\to\infty} v_a(a,b),\ \lim_{b\to\infty} v_b(a,b)\ <\ 1$$

and

$$v_a(0,b)\ >\ 1,\ \ v_b(a,0)\ >\ 1\ ,$$

The Nöldeke-Schmidt Model

- first best investments:

$$(a^*, b^*) \in \arg\max_{a,b} v(a,b) - a - b \gg (0,0)$$

  are uniquely defined by the FOCs:

$$v_a(a^*, b^*) = v_b(a^*, b^*) = 1\ .$$

- efficient investment of B given a:

$$v_b(a, b^*(a)) = 1\ .$$

The Nöldeke-Schmidt Model

The contract specific environment

At date 0 only contracts on the allocation of ownership rights on K are feasible. Examples:

- A ownership, B ownership,
- joint ownership,
- options on ownership: A owns K, but B has the right to buy K at price p at date t.

The owner of K can prevent the other party from using the asset.

At date 3 the parties can write a complete contract in order to realize and share the surplus $v(a, b)$:

- They will always realize all of $v(a, b)$.
- The surplus will be shared in proportion $(\lambda, 1 - \lambda)$.
- Threat point of the renegotiation game depends on the ownership structure and the type of investments.

Threatpoints

If one party has the ownership right on K, it can prevent the other party from using the asset.

- Joint ownership: $\Rightarrow$ $(0, 0)$.
- A-Ownership: $\Rightarrow$ $(v(a, \beta b), 0)$
  A can realize $v(a, \beta b)$ on its own, B gets 0.
- B-ownership: $\Rightarrow$ $(0, v(\alpha a, b))$
  A gets 0, B can realize $v(\alpha a, b)$ on its own.

$\alpha, \beta \in [0, 1]$ reflect the nature of the investments:

$\alpha = 1$:	A's investment is in **physical capital**, i.e., B can make full use of it even without A's cooperation.
$\alpha = 0$:	A's investment is in **human capital**, i.e., without A, the investment is useless to B.

$\alpha, \beta \in (0, 1)$ allows for less extreme cases.

Payoffs at date 3:

Let $o \in \{A, B, J\}$ denote the ownership structure. Then final payoffs are given by

$$U^A(a, b \mid o) = \begin{cases} v(a, \beta b) + \lambda[v(a,b) - v(a, \beta b)] - a & \text{if } o = A \\ \lambda[v(a,b) - v(\alpha a, b)] - a & \text{if } o = B \\ \lambda v(a,b) - a & \text{if } o = J \end{cases}$$

and

$$U^B(a, b \mid o) = \begin{cases} (1-\lambda)[v(a,b) - v(a, \beta b)] - b & \text{if } o = A \\ v(\alpha a, b) + (1-\lambda)[v(a,b) - v(\alpha a, b)] - b & \text{if } o = B \\ (1-\lambda) v(a,b) - b & \text{if } o = J \end{cases}$$

Option Contracts without Renegotiation

Proposition 6.5

No unconditional allocation of ownership rights induces first best investments.

Remarks:

1. Proposition 6.5 adapts the basic result of Grossman and Hart (1986) to our setting with sequential investments.
2. Intuition: It is impossible to give both parties the full marginal return of their investments simultaneously.
3. The proof is slightly more complicated than in Grossman and Hart, because A can affect B's investment incentives through her investment decision.

Option Contracts without Renegotiation

Options on ownership rights:
A owns K, but B has the option to buy K at price p at date $t = 2.5$.

Assumption 6.4

For all a, B's optimal investment decisions satisfy

(a) $b(a \mid B) = b^*(a)$
(b) $b(a \mid A) = 0$

Assumption 6.4 requires that

- B, when owning the asset, makes the conditionally efficient investment
- B does not invest under A-ownership

Assumption 6.4 will be imposed throughout the paper. It will be discussed in more detail below.

Option Contracts without Renegotiation

Proposition 6.6

An option contract, giving B the right to buy the asset at price

$$p^* = U^B(a^*, b^* \mid B)$$

after both investments have been made, induces first best investment levels (a^*, b^*).

Option Contracts without Renegotiation

Intuition:

- B's valuation of the asset increases with A's investment. Thus, it is worth B's while to exercise his option only if A invested enough. By choosing p^* appropriately, B will exercise his option if and only if A invested at least a^*.
- If A invests $a < a^*$, he remains the owner and B will not invest (by Ass. 6.4(b)). Therefore, the asset is worth very little to A. Hence, A prefers to choose $a = a^*$.
- If A invests $a > a^*$, B will exercise his option and receive most of the marginal benefits of A's investment. Therefore A will not invest too much.
- Given that A chooses $a = a^*$ and B becomes the owner, it is optimal for B to invest efficiently, too, by Ass. 6.4(b).

Remark: The result carries over if investments are multi-dimensional.

Option Contracts without Renegotiation

Proof: Suppose A chooses a^*. If B does not want to exercise the option, he should not invest (by Ass. 6.4(b)) and receive an overall payoff of 0. On the other hand, if he is going to exercise his option, he should invest efficiently (by Ass. 6.4(a)) and receive

$$u^B(a^*, b^*) \;=\; U^B(a^*, b^* \mid B) - p^* \;=\; 0 \,.$$

Hence, given a^* it is optimal for B to invest efficiently and to exercise his option. The resulting payoff for A is given by

$$u^A(a^*, b^*) \;=\; p^* + U^A(a^*, b^* \mid B) \;=\; S(a^*, b^*)$$

Note that B can always guarantee himself a payoff of at least 0 (by not investing and not exercising his option). Hence, given that A invested a, B's continuation payoff, $u^B(a)$, satisfies $u^B(a) \geq 0$. If A chooses $a \neq a^*$ her continuation payoff, $u^A(a)$, thus satisfies

$$u^A(a) \;\leq\; u^A(a) + u^B(a) \;\leq\; \max_b S(a, b) \;<\; S(a^*, b^*) \,.$$

Hence, $a = a^*$ is the uniquely optimal choice for A. *Q.E.D.*

Discussion of Assumption 6.4:

- Ass. 6.4 holds if both parties invest in physical capital, i.e. if $\alpha = \beta = 1$. In this case payoffs are given by

$$U^A(a, b \mid o) = \begin{cases} v(a,b) - a & \text{if } o = A \\ -a & \text{if } o = B \end{cases}$$

$$U^B(a, b \mid o) = \begin{cases} -b & \text{if } o = A \\ v(a,b) - b & \text{if } o = B \end{cases}$$

- Ass. 6.4(a) is also satisfied if investments are independent, i.e.

$$v(a, b) = u(b) - c(a) ,$$

  i.e., if A's investment lowers the cost of producing the output, whereas B's investment increases his valutation of the output.

Option Contracts without Renegotiation

- Ass. 6.4(b) is also satisfied if *B*'s bargaining power is sufficiently low or if the physical capital component of his investment (β) is sufficiently large. It holds if

$$(1-\lambda)(1-\beta)v_b(a,0) \;<\; 1 \;.$$

- Ass. 6.4(a) is necessary for simple option contracts to work; Ass 6.4(b) is much too strong, but greatly facilitates the exposition.

Option Contracts with Renegotiation

The parties can renegotiate the ownership structure if this yields an efficiency improvement.
Suppose A chooses $a < a^*$. Given the option contract of Proposition 6.6, B would not exercise the option and not invest efficiently.

$\Rightarrow$ p^* will be renegotiated to make sure that B does exercise his option.

$\Rightarrow$ If A gets some fraction of the surplus from renegotiation, her payoff from underinvesting goes up as compared to the no-renegotiation case.

$\Rightarrow$ A may have an incentive to invest too little.

Option Contracts with Renegotiation

Proposition 6.7

An option contract that gives B the right to buy the asset at price

$$p^* = U^B(a^*, b^* \mid B)$$

at date $2\frac{1}{2}$ *induces first best investment levels* (a^*, b^*) *with renegotiation.*

Option Contracts with Renegotiation

Intuition:

- Suppose A invests $a = a^*$. In this case B will exercise his option anyway even if there is no renegotiation, and B will invest efficiently. Hence, if A refuses to renegotiate, she will get

$$U^A(a^*, b^*) \; = \; p^* + U^A(a^*, b^* \mid B) \; = \; S(a^*, b^*) \; .$$

- On the other hand, B cannot get less than 0, since he can always choose $b = 0$, not exercise his option, and refuse to renegotiate.
- Hence, A cannot get more than $S(a^*, b^*)$ and a^* is again her optimal investment level.

Option Contracts with Renegotiation

Note that if renegotiation is possible, Proposition 6.5 need not hold anymore: A-ownership, that is renegotiated to B-ownership at date $1\frac{1}{2}$ may induce both parties to invest efficiently.

Proposition 6.8

With renegotiation an initial contract specifying A-ownership induces first best investment levels if and only if $v_a(a^*, 0) = 1$.

This condition is satisfied if investment incentives are independent ($v_{ab}(a, b) = 0$). In all other cases the simple ownership arrangement considered here does not lead to efficiency.

Conclusions

1. Options on ownership rights can implement the first best if investments are sequential and if Assumption 6.4 is satisfied.
2. With simultaneous investments option contracts cannot implement the first best in general, but they do achieve a more efficient allocation of ownership rights as compared to unconditional ownership structures.
3. Assumption 6.4(a) can be relaxed if we allow for more complicated option contracts.
4. Results are robust to the introduction of (some) uncertainty.