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Convertible Securities and Venture Capital Finance

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ABSTRACT

This paper offers a new explanation for the prevalent use of convertible securities in venture capital finance. Convertible securities can be used to endogenously allocate cash-flow rights as a function of the state of the world and the entrepreneur's effort. This property can be used to induce the entrepreneur and the venture capitalist to invest efficiently into the project. The result is robust to renegotiation and to changes in the timing of investments and information flows. The model is consistent with the observations that conversion is often automatic and that convertible securities are rarely used by outside investors.

Introduction

VENTURE CAPITAL ACCOUNTS FOR ONLY a tiny fraction of total corporate investment in the United States, but it has had a dramatic impact on economic growth and the creation of new jobs since the 1970s. Microsoft, Intel, Apple, Federal Express, Cisco Systems, Genentech, and many other icons of high technology were all venture-capital backed in their early stages. However, the financing of young entrepreneurial firms is prone to severe incentive problems. To deal with these problems, venture capital firms have developed sophisticated contracting practices, some of which are unique to the venture capital industry. In particular, the purchase of convertible securities by the venture capitalist is by far the predominant form of investment.[1] This is surprising because convertible securities

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[1] In the sample of Kaplan and Strömberg (2000, p. 13) convertible securities were used in 189 out of 200 financing rounds. Similar empirical findings are reported by Sahlmann (1990) and Gompers (1997).

are very rarely used by banks or other outside investors who finance the bulk of small (but more established and less risky) companies.[2]

This paper offers a new explanation for the prevalent use of convertible securities in venture capital finance. The starting point of the analysis is the observation that the ultimate success of high-potential, entrepreneurial firms depends not only on the quality of the project and the effort provided by the entrepreneur, but also on the commitment of the venture capitalist. It is a well-documented fact that venture capitalists provide not only the necessary financial means to develop the project, but that they are also actively involved in the management of the firms they finance.[3] Venture capitalists are typically well connected in the specific industry, they help to recruit key personnel, they negotiate with suppliers and customers, they advise the entrepreneur on strategic decisions, they play a major role in structuring mergers, acquisitions, and initial public offerings, and sometimes they are even involved in the day-to-day operations of the firm. If things turn sour, venture capitalists often replace the founder of the company by a professional CEO and/or sell off or liquidate the firm.[4]

Our model focuses on the incentive properties of convertible securities. First, we point out a specific feature of convertible securities that provides a powerful incentive mechanism in a sequential double moral hazard problem, which can be used to induce both parties to invest efficiently without making both of them full residual claimant on the margin. The incentive mechanism exploits the fact that the venture capitalist will invest only if he exercises his conversion rights, and that he will convert only if the entrepreneur worked sufficiently hard, which, in turn, induces the entrepreneur to put in the efficient amount of effort. A suitably chosen convertible security strictly outperforms any standard-debt equity contract. Second, it is shown that convertible securities implement efficient investments only if the contribution of the venture capitalist to the project is sufficiently important. The active involvement of the investor is a characteristic feature of venture capital finance. Thus, the model can explain both why convertible securities are so popular in venture capital finance, and also why convertible securities are uncommon if banks or other passive investors finance small companies. Finally, the implications of our model are consistent with the empirical findings on VC finance. In particular, it can account for the fact that many contracts have convertible securities with an exercise date at the time of the IPO and/or automatic conversion clauses; it is consistent with Kaplan and Strömberg's (2000) finding that the share of cash flows that goes to the venture capitalist is decreasing with the performance of the company, and it is consistent

[2] We restrict attention to the case where there is only one or a few key investors for a project. Convertible securities are also issued by large corporations to dispersed investors. In these cases, convertible securities yield tax benefits and may have other advantages that are beyond the scope of our model.

[3] See, for example, Sahlmann (1990, p. 508), Lerner (1995), or Hellmann and Puri (2000, 2002).

[4] Gorman and Sahlman (1989) report that, on average, each venture capitalist is responsible for 10 firms, that he visits each firm 19 times per year, and that he spends 100 hours annually at each firm.

with the prevalent use of stage financing and syndication of venture capital investments.

There are several other papers that deal with optimal contract design for an inside investor. Admati and Pfleiderer (1994) analyze stage financing and show that a "fixed fraction contract" that gives the venture capitalist a fixed fraction of the equity of the company in all financing rounds induces the inside investor to make optimal investment decisions and not to misprice securities in later financing rounds. Repullo and Suarez (1999) consider a double moral hazard problem between the entrepreneur and a venture capitalist. They show that the venture capitalist should get no compensation for his initial investment in the lower tail and high compensation in the upper tail of the distribution of returns. They demonstrate that this sharing rule can be approximated by the use of warrants. Bergemann and Hege (1997) have a dynamic moral hazard model in which the venture capitalist learns the quality of the project over time and has to decide when to get out. They show that the optimal contract is a mixture of debt and equity and that the entrepreneur's share of any proceeds decreases over time. None of these papers explains the predominant use of convertible securities.

A number of other roles for convertible debt in resolving agency and information problems have been suggested in the literature. Green (1984) points out that convertible securities can be used to mitigate the problem of excessive risk taking of the entrepreneur that arises if debt is used. The idea is that a convertible security reduces the entrepreneur's payoff for very good profit realizations and thus makes excessive risk taking less attractive. However, in his model, the same incentive effect can be achieved by using a combination of standard debt and equity.[5] Our model complements Green by focusing solely on the effort incentives for the entrepreneur and the investor. Cornelli and Yosha (1997) focus on the entrepreneurial incentives to engage in "window dressing" in order to induce the venture capitalist to finance the second stage of the project. With a convertible debt contract, this signal manipulation is less profitable, because the venture capitalist will convert his debt into equity if the firm looks too good, which reduces the entrepreneur's profit.[6]

Finally there is a branch of the literature that focuses on conflicts of interests between entrepreneurs, venture capitalists, and outside financiers that stem from nontransferable private benefits of control and affect critical decisions such as the liquidation of the venture (Marx (1998)) or the sale to another company or an IPO (Berglöf (1994)). In these models, convertible securities are used to allocate control rights to the right persons in different states of the world. However, Gompers (1997) and Hellmann (1998) argue that the allocation of cash-flow rights can be separated from the allocation of control rights by the use of covenants.

[5] See also Biais and Casamatta (1999), who use stock options to reduce excessive risk taking.

[6] Similarly, D'Souza (2001) develops a model in which convertible securities can be used as a mechanism to induce the entrepreneur to truthfully reveal the state of the world to the venture capitalist.

Gompers documents that covenants are indeed frequently used to give the venture capitalist the right to control the board of directors, to approve major expenditures, to liquidate the firm, and even to replace the entrepreneur by an outside manager. Typically, the venture capitalist is given these contractual rights independently of the financial structure of the company.[7] For this reason, it can be argued that the most important function of convertible securities is to allocate cash-flow rights, while the allocation of control rights can be achieved separately through the use of covenants.

Our paper is also related to the literature on incomplete contracts and the optimal allocation of ownership rights. Grossman and Hart (1986) argue that the allocation of ownership rights matters if only incomplete contracts can be written. Nöldeke and Schmidt (1998) show that it may be efficient to use a "conditional ownership structure" in the Grossman–Hart model, which can be implemented by using "options on ownership rights" that play a similar role to the convertible securities considered here. The present paper generalizes Nöldeke and Schmidt in several respects. It allows for an upfront investment that has to be financed and for "large" uncertainty that is characteristic for venture capital. In Nöldeke and Schmidt, both parties are assumed to have unlimited wealth, while in this paper, the entrepreneur is wealth constrained and protected by limited liability. Finally, their "option to own" is an all-or-nothing decision, while in this paper, the venture capitalist may get the option to convert his debt claim into some fraction $\alpha < 1$ of the equity of the firm.

The rest of the paper is organized as follows. Section I analyzes the incentive properties of a convertible security in a general framework with sequential investments. These results are applicable to any situation in which the active involvement of two parties is required for the ultimate success of a joint venture. However, the convertible securities employed in this section give one party the option to get all of the equity of the firm if he exercises his conversion right. This is not what is typically observed in venture capital finance. Therefore, in Section II, the general model is adapted to the context of venture capital investments. Here, we assume that the entrepreneur is wealth constrained and protected by limited liability and that the venture capitalist's contribution is most important if the project fails or if the project is very successful, but not if it turns out mediocre. We demonstrate how the entrepreneur and the venture capitalist can both be induced to invest efficiently in all states of the world, even if the venture capitalist can convert his debt claim only in some fraction $\alpha < 1$ of the firm's equity. Furthermore, it is shown that our results are consistent with the empirical observations on venture capital contracts. Section III offers several extensions and variations of the basic model, including multiple claims held by the venture capitalist, syndication, stage financing, multidimensional effort by the entrepreneur, and changes in the timing of information flows

[7] This has been confirmed by Kaplan and Strömberg (2000, p. 93), who conclude that "(T)he distinguishing characteristic of VC financings is that they allow VCs to separately allocate cash-flow rights, voting rights, board rights, liquidation rights, and other control rights."

and investments. Our main results are robust to these changes of the model. Section IV concludes.

I. The Incentive Properties of Convertible Securities

In this section, we consider a general double moral hazard problem with sequential investments and show that a convertible security can induce both parties to invest efficiently even though only one party is going to be residual claimant of social surplus on the margin. This model extends a model by Nöldeke and Schmidt (1998) in several directions. First, we introduce an upfront investment that has to be financed by one of the parties. Second, it is shown that the surplus can be shared in any desired fashion between the two parties. Finally, we introduce "large" uncertainty and show that the first best can still be implemented if the parties can renegotiate the initial contract after the realization of the state of the world has been observed.

A. The Basic Model

There are two players who can realize a potentially profitable project. In the next section, we will specialize the model to venture capital finance. This is why we call the two players "entrepreneur" (E) and "venture capitalist" (VC), but for the general model of this section, the restriction to venture capital is not necessary. The project requires a fixed initial investment $I > 0$ and generates a return, $v(a, b, \theta)$ that depends on three factors: the effort that is provided by the entrepreneur, a, the effort and/or further financial investment of the venture capitalist, b, and the realization of the "state of the world," θ, that is, the quality of the project, the ability of the entrepreneur, market conditions, and so forth.

The relationship between E and VC is modeled as follows. At date 0, E and VC have to negotiate a contract that governs their relationship. At that stage, the state of the world is unknown to both parties. We assume that it can be observed by both of them only after the initial investment I has been sunk, but it is not verifiable to the courts and cannot be contracted upon at date 0. There may be many potential venture capitalists and many potential entrepreneurs ex ante and competition between them may affect how the contracting parties share the potential surplus between them, but after the initial investment I has been sunk, VC and E are locked in with each other. At date 1, E has to make a relationship specific investment, $a \in R_0^+$, to develop the project. This investment cannot be contracted upon and is best thought of as the effort E invests into the firm. At date 2, the venture capitalist has to decide on his engagement for the project.[8] VC's action $b \in R_0^+$ cannot be contracted upon either. Both investments are measured by their costs. Finally, at date 3, the gross surplus $v(a, b, \theta)$ that can be generated by the project is realized and split between the two parties according to

[8] The sequential nature of the investments and different assumptions about the timing of the investments and the information flows are discussed in Sections III.E and F.

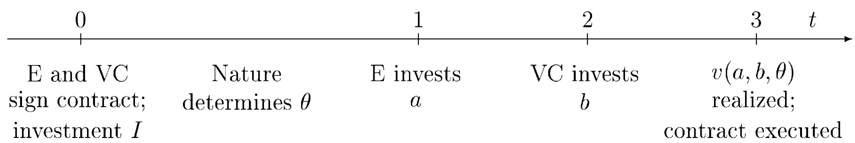

Figure 1. Time structure of the model.

the initial contract that had been signed at date 0. The net total surplus is given by $S(a, b, \theta) = v(a, b, \theta) - a - b$. Both parties are assumed to be risk neutral.

The time structure of the model is summarized in Figure 1.

As a reference point, let us define the first best efficient investment levels. Let

$$S(a(\theta), b(\theta), \theta) = v(a(\theta), b(\theta), \theta) - b(\theta) - a(\theta) \tag{1}$$

be the social surplus in state θ if E chooses a (θ) and VC chooses b (θ). Efficient investment levels are assumed to be unique and given by

$$(a*(\theta), b*(\theta)) = \arg\max_{a,b} v(a, b, \theta) - b - a. \tag{2}$$

We want to show that a suitably chosen convertible security implements the efficient investment choices. For this purpose, it is sufficient to restrict attention to a convertible security (C, F), which gives the following option to the venture capitalist. At some date t, which is specified in the contract, VC can choose either to receive the fixed payment C or to pay the additional amount F and to convert his debt into 100 percent of the equity of the venture. Thus, given (C, F), payoffs are

$$U^E = \begin{cases} v(a, b, \theta) - C - a & \text{if VC does not convert} \\ F - a & \text{if VC converts} \end{cases} \tag{3}$$

$$U^{VC} = \begin{cases} C - b - I & \text{if VC does not convert} \\ v(a, b, \theta) - F - b - I & \text{if VC converts} \end{cases} \tag{4}$$

The initial contract will be renegotiated by the two parties whenever a possibility for an efficiency improvement arises. In particular, renegotiation may be beneficial after the realization of the state of the world and/or after E has made her investment decision. We assume that the surplus from renegotiation is split in proportion λ, $(1 - \lambda)$ between E and VC.

B. Convertible Securities without Uncertainty

To better understand the incentive properties of a convertible security, let us first ignore the uncertainty about the realization of the state of the world. Suppose that θ is given and that

$$S(a*(\theta), b*(\theta), \theta) \geq I, \tag{5}$$

so total social surplus is sufficient to cover the upfront investment cost I. Without loss of generality, we assume that the initial investment is paid for by VC. The following proposition is an extension of Proposition 3 of Nöldeke and Schmidt (1998). It shows that a suitably chosen convertible security implements ($a^*(\theta)$, $b^*(\theta)$), and that the surplus of the project can be shared between the two parties in any desired fashion.

PROPOSITION 1: *For any given θ, a convertible security (C, F) with*

$$C = I + K \tag{6}$$

$$F = v(a^*(\theta), b^*(\theta), \theta) - b^*(\theta) - I - K \tag{7}$$

where

$$0 \leq K \leq S(a^*(\theta), b^*(\theta), \theta) - I, \tag{8}$$

that gives VC the right at date 3 either to be paid back C or to pay an additional amount F and convert his debt into 100 percent of the equity of the firm induces both parties to take the efficient actions $a^*(\theta)$ and $b^*(\theta)$ and gives the following payoffs to the two parties:

$$U^E = S(a^*(\theta), b^*(\theta), \theta) - I - K \tag{9}$$

$$U^{VC} = K. \tag{10}$$

The intuition for this result is as follows: To induce VC to choose $b = b^*(\theta)$, it is necessary that VC holds all the equity of the company. However, he will get the equity only if he chooses to exercise his conversion option. The crucial point to note is that the value of his conversion option depends on E's investment. The more E invests, the more attractive it is for VC to convert. By choosing the exercise price of the option appropriately, VC is just indifferent between "investing $b = b^*(\theta)$ and converting," and "investing $b = 0$ and not converting" if E invested $a^*(\theta)$. In both cases, his payoff is just $C - I = K$. Note that F has been chosen such that if E invested $a^*(\theta)$ and VC converts, then E gets the maximum social surplus minus a fixed payment that she would have to make independently of whether the option is exercised or not. Investing more than $a^*(\theta)$ cannot be profitable for E, because VC would still exercise his conversion option and receive all of the marginal returns of E's additional investment. Investing less than $a^*(\theta)$ cannot increase E's payoff either. In this case F will be renegotiated and reduced in order to make sure that VC gets to own 100 percent of the equity of the firm and chooses b efficiently. However, VC must get at least $C - I = K$, which he can guarantee himself by choosing $b = 0$ and not converting. Thus, VC's payoff cannot be smaller than if E invested $a^*(\theta)$. But total social surplus is reduced (as compared to $a^*(\theta)$), so E's payoff must be smaller, too. Hence, E is induced to choose exactly the first best investment level $a^*(\theta)$, VC chooses $b^*(\theta)$ and exercises his conversion option, and there is no renegotiation on the equilibrium path.

The most interesting feature of a convertible security is that it can induce E and VC to invest efficiently *without making both parties residual claimant on the* *margin*, that is, without using a "budget breaker" (Holmström, 1982). Furthermore, the argument given above does not rely on any differentiability assumptions. Even if investments are discrete or multidimensional, a suitably chosen convertible security implements the first best. It is these features that make the convertible security a powerful incentive device in a double moral hazard context.[9]

The question arises whether a convertible security is the simplest contract that implements efficient investments or whether the same outcome can be achieved by using a standard debt–equity contract, possibly with renegotiation. Nöldeke and Schmidt (1998, Proposition 4) show that this is possible if and only if either the entrepreneur has all the bargaining power in the renegotiation game ($\lambda = 1$) or if investments are independent on the margin. To see why, suppose that the initial contract gives 100 percent of the equity to E. After E's investment has been made, this contract will be renegotiated to give 100 percent of the equity to VC in order to induce him to invest efficiently. Thus, E's payoff (ignoring a possible debt payment) is given by

$$U^E = v(a, 0, \theta) - a + \lambda[v(a, b^*(a, \theta), \theta) - b^*(a, \theta) - a - v(a, 0, \theta) + a] \,, \tag{11}$$

where $b^*(a, \theta)$ is the efficient investment of VC given that E has chosen a in state θ. Using the envelope theorem, the marginal return to E's investment is

$$\frac{\partial U^E}{\partial a} = (1 - \lambda)\,\frac{\partial v(a, 0, \theta)}{\partial a} + \lambda\,\frac{\partial v(a, b^*(a, \theta), \theta)}{\partial a} - 1. \tag{12}$$

If $\lambda < 1$, we have a standard hold-up problem that distorts E's investment incentives, except for the special case where $\partial v(a, 0, \theta)/\partial a = \partial v(a, b^*(a, \theta), \theta)/\partial a$. Hence, in general, there does not exist a debt–equity contract that implements the first best, while a properly designed convertible security does.

C. Convertible Securities with Uncertainty

Suppose now that the parties have to write the initial contract before they know the realization of θ. At date 0, both parties only know the cumulative distribution function $G(\theta)$. Suppose that the parties write an initial contract with F very large so that VC would never exercise his conversion option. This contract is equivalent to a pure debt contract. If there was no renegotiation, VC would get the fixed payment C and thus choose $b = 0$ at date 2, independent of E's investment. Thus, without renegotiation, E would choose

$$\hat{a}(\theta) = \arg\max v(a, 0, \theta) - a. \tag{13}$$

However, after the realization of θ, there is scope for an efficiency improvement by lowering F such that VC would exercise his conversion option and invest efficiently if and only if E invests $a^*(\theta)$. The following proposition shows that with

[9] At this point, it is interesting to compare the mechanism described here to the mechanism analyzed by Repullo and Suarez (1999). Repullo and Suarez also analyze a double moral hazard problem. However, they consider the case of simultaneous investments and look for a second best optimal sharing rule that gives optimal *marginal* incentives to both players.

renegotiation, the first best can again be implemented and that it is possible to share the maximum expected social surplus in any desired way.

PROPOSITION 2: *Suppose the parties have written a pure debt contract with*

$$C = I - (1-\lambda)\int_{\theta}[S(a^*(\theta), b^*(\theta), \theta) - v(\hat{a}(\theta), 0, \theta) + \hat{a}(\theta)]\mathrm{d}G(\theta) + K \qquad (14)$$

where $0 \leq K \leq \int_{\theta} S(a^*(\theta), b^*(\theta), \theta)\mathrm{d}G(\theta) - I$ *and* $\hat{a}(\theta)$ *is given by* (13). *This contract implements first best investment decisions with renegotiation and yields expected payoffs*

$$EU^E = \int_{\theta} S(a^*(\theta), b^*(\theta), \theta)\mathrm{d}G(\theta) - I - K \qquad (15)$$

$$EU^V = K \qquad (16)$$

On the equilibrium path, this contract will be renegotiated to a convertible security (C, F), *where* C *is given by* (14) *and*

$$F = v(a^*(\theta), b^*(\theta), \theta) - b^*(\theta) - C. \qquad (17)$$

Propositions 1 and 2 highlight the incentive properties of convertible securities. They offer a new rationale for the use of convertible securities in addition to risk sharing and tax explanations that are commonly referred to in the literature. However, when we want to apply this explanation to the specific context of venture capital finance, where convertible securities are frequently used, Propositions 1 and 2 are not quite satisfactory.

II. Convertible Securities in Venture Capital Finance

Convertible securities are the predominant form of investment in young entrepreneurial firms by venture capitalists (see footnote 1). However, convertible securities are very rarely used by banks or passive outside equity holders who finance the bulk of small (but more established and less risky) companies. It is often emphasized that, in contrast to banks, venture capitalists play a much more active role in many of the firms they finance, and that the contribution of the venture capitalist is of crucial importance to the ultimate success of the venture. Therefore, the initial contract that governs the relationship between the entrepreneur and the venture capitalist has to induce both parties to invest efficiently into the project. This fits the setup of the model of Section I.

However, the actual contracts that we observe in the venture capital industry differ in several important respects from the optimal contracts characterized in Section I. First, VCs typically do not have the option to get 100 percent of the equity of the firm, but rather some fraction that is considerably smaller than one. Second, the model of Section I implies that the conversion option will always be exercised on the equilibrium path (possibly after renegotiation). In the venture capital industry, however, the conversion option is exercised only for the most profitable ventures. Many less profitable ventures either pay back their debt or are liquidated by VC. Finally, entrepreneurs of young start-up companies are typically wealth constrained and protected by limited liability. Thus, they may default on their debt, which may distort investment incentives.

On the other hand, in the specific context of venture capital finance, there are several stylized facts that can be used to impose more structure on the model. Employing this additional structure will allow us to show that the type of convertible securities observed in the VC industry are indeed very well suited to solve the double moral hazard problem between the entrepreneur and the venture capitalist.

A. Empirical Findings about Venture Capital Finance

At the contracting stage between E and VC, there is typically a high degree of uncertainty about the prospects of the project. It is often unclear whether the project is technologically feasible, whether there will be competitors offering superior or less expensive products, whether the right employees can be attracted, or whether the entrepreneur is capable of running the company. In a sample of 383 VC investments analyzed by Sahlmann (1990, p. 484), about 35 percent of all projects yielded a total loss or were unable to repay the initial investment. Roughly 15 percent of all projects in this sample (called "high flyers") were highly profitable, yielding a return of more than five times the initial investments. The rest (or roughly 50 percent) were moderately successful. This last type of project is called "living dead" in the industry. Even though these ventures are moderately profitable, VCs often consider them unworthy of additional investments and do not want to spend more time or money on them than absolutely necessary.[10]

But, of course, the profitability of the project does not depend only on exogeneous factors. The entrepreneur's effort is obviously important. She has to build up her company, she has to engage in R&D and to develop the product, she has to set up production facilities and market her product, and so forth.[11] Furthermore, for the ultimate success of many young entrepreneurial firms, the active support of an experienced venture capitalist is also of crucial importance. VCs tend to have deep industry-specific knowledge, they are well connected within the industry, and they play an active and important role in many of the ventures they finance.[12] In a recent empirical study, Hellmann and Puri (2002) show that

[10] See Gorman and Sahlman (1989, p. 237).

[11] Typically, the problem is not to get the entrepreneur to work hard enough, but rather to induce her to allocate her effort efficiently. For example, many start-up entrepreneurs have a background as engineers or scientists. Thus, they may spend too much effort on, say, additional R&D rather than on marketing, controlling, or personnel. In our model, there is no problem allowing for multidimensional investment or effort decisions, which is shown in Section III.D.

[12] This is by now a fact well documented in the empirical literature. See, for example, Gorman and Sahlman (1989) or Kaplan and Strömberg (2000). Hellmann and Puri (2000) compare companies that are venture capital financed to similar firms that are not backed by venture capital and show that venture capitalists do have a significant impact. Venture-capital-backed firms are faster to bring their products to market. Kortum and Lerner (2000) report that the amount of venture activity in an industry significantly increases the rate of patenting.

two roles of VCs can be distinguished. First, VCs support the entrepreneur if the company is on the right track: They give advice on strategic decisions, they help to find key employees and to design suitable compensation packages for them, they get in contact with potential suppliers and customers, and they may even get involved in the day-to-day operations of the firm. These "soft" or "supportive" actions are *complements* to E's efforts. They are privately costly to the VC but benefit the company and the entrepreneur. Second, if things turn sour, VCs exercise their control rights. In particular, they liquidate unprofitable companies and/or replace the original founder with a professional outside CEO. These "hard" or "control" actions are also costly to the VC and enhance the value of the company, but they are in conflict with the entrepreneur, take away her private benefits from running the company, and are *substitutes* to E's effort.[13]

Finally, in Section I, we assumed that E is not wealth constrained and always able to repay her debt. However, the founders of young entrepreneurial firms are typically wealth constrained and protected by limited liability, so they cannot offer any collateral that can be used to secure VC's investment except for the returns of the venture.

B. Adaptation of the Model to Venture Capital Finance

Following Sahlman (1989) and Hellmann and Puri (2000, 2002), we distinguish three different states of the world, $\theta \in \{\underline{\theta}, \theta_m, \overline{\theta}\}$, $\underline{\theta} < \theta_m < \overline{\theta}$, and three corresponding actions of the venture capitalist, $b \in \{b_c, b_0, b_s\}$. If $\theta = \underline{\theta}$, then the expected returns of the project are very poor and it should not be run by E. In this case, it is efficient that VC chooses action $b = b_c$, that is, he should exercise his control rights, get rid of the entrepreneur and liquidate or sell off the assets of the firm. If $\theta = \overline{\theta}$, then the firm is a "high flyer," which is potentially very profitable. In this case, it is efficient that VC chooses action $b = b_s$, that is, he should actively support the entrepreneur and complement her efforts to make the firm a success. Finally, if $\theta = \theta_m$, the firm is mediocre, a "living dead" as it is called in the industry. In this case, the project may be able to return the initial investment, but it is not worth much additional effort by the venture capitalist, so VC should choose $b = b_0$, where $0 = b_0 < b_c,\ b_s$. Thus, b_0 should be interpreted as the default action of VC (no further financial investment and as little effort as possible). Note that we normalize VC's cost of taking action b_0 to 0. The ex ante probability of the good state is p, of the medium state is q, and of the bad state is $1 - p - q$, with $0 < p,\ q,\ 1 - p - q < 1$.

This is stated formally in the following assumption.

[13] Hellmann and Puri (2002) report that "support" and "control" seem to be mutually exclusive actions. They divide their sample in two subsamples, those firms that did experience a CEO turnover and those firms that did not. In the subsample of firms without turnover, they find a significant impact of the venture capitalist on team building at the level below the CEO (supportive action). However, this is not true in the subsample of firms that experienced a turnover, where the venture capitalist seems to devote all his time and attention to replacing the founder with a professional CEO.

ASSUMPTION 1: *The gross surplus function* $v(a, b, \theta)$ *is twice continuously differentiable, strictly increasing and strictly concave in* a, *and strictly increasing in* θ *for all* $a \in \mathbb{R}_0^+$, $b \in \{b_c, b_0, b_s\}$ *and* $\theta \in \{\underline{\theta}, \theta_m, \overline{\theta}\}$. *All investments are measured by their costs. Furthermore,*

(a) $v(a, b_c, \underline{\theta}) - b_c = \underline{v} \geq v(a, b, \underline{\theta}) - b$ *for all* $a \in \mathbb{R}_0^+$ *and* $b \in \{b_0, b_s\}$.
(b) $v(a, b_0, \theta_m) - 0 > v(a, b, \theta_m) - b$ *for all* $a \in \mathbb{R}_0^+$ *and* $b \in \{b_c, b_s\}$.
(c) $v(a, b_s, \overline{\theta}) - b_s \geq v(a, b, \overline{\theta}) - b$ *for all* $a \in \mathbb{R}_0^+$ *and* $b \in \{b_0, b_c\}$.
(d) $\frac{\partial v(a,b',\theta')}{\partial a} - \frac{\partial v(a,b,\theta)}{\partial a} > 0$ *for all* $a \in \mathbb{R}_0^+$, $\theta' \geq \theta$, $\theta', \theta \in \{\theta_m, \overline{\theta}\}$, $b' \geq b$, *and* $b', b \in \{b_0, b_s\}$.

Assumption 1(a) to (c) simply say that it is efficient for VC to choose

$$b^*(\theta) = \begin{cases} b_s & \text{if } \theta = \overline{\theta} \\ b_0 & \text{if } \theta = \theta_m \\ b_c & \text{if } \theta = \underline{\theta} \end{cases} \tag{18}$$

Assumption 1(d) says that a and b_s are complements at the margin, that is, not only total surplus but also marginal surplus with respect to a increases if VC chooses $b = b_s$ rather than $b = b_0$. Note that Assumption 1(a) implies that in the bad state, a and b_c are substitutes at the margin: If VC chooses $b = b_c$ rather than $b \in \{b_0, b_s\}$, then the marginal surplus with respect to a is reduced to 0. Furthermore, Assumptions 1(a) and (d) imply $0 < a^*(\theta_m) < a^*(\overline{\theta})$.

Furthermore, we have to assume that

$$S(a^*(\theta_m), b^*(\theta_m), \theta_m) = v(a^*(\theta_m), b_0, \theta_m) - a^*(\theta_m) - b_0 > \frac{I - \underline{v}}{p + q} + \underline{v}. \tag{19}$$

This assumption is due to the fact that E is protected by limited liability and cannot be forced to pay more than $\underline{v}$ is the bad state of the world. Suppose that VC holds a debt claim $D = \frac{I-\underline{v}}{p+q} + \underline{v}$. Then, net total surplus in the medium state (and thus in the good state, too) is sufficient to repay the debt and VC gets an expected payoff of $(p + q)D + (1 - p - q)\underline{v} = I$. Hence, in expectation, this return is just sufficient to cover his initial investment cost I.

PROPOSITION 3: *Consider a convertible security* (C, K, α) *with*

$$C = \frac{I - \underline{v}}{p + q} + \underline{v}, \tag{20}$$

$$\alpha = \frac{C + b_s}{v(a^*(\overline{\theta}), b_s, \overline{\theta})} < 1, \tag{21}$$

and

$$0 \leq C + K \leq S(a*(\theta_m), b*(\theta_m), \theta_m) \tag{22}$$

which gives VC the option to choose at date 3 whether to be repaid C+K or to be repaid K and convert C into fraction α of the equity of the company. This convertible security implements first best investment decisions with renegotiation if

$$\lambda[v(a, b_s, \overline{\theta}) - b_s - v(a, b_0, \overline{\theta})] - a \ \leq \ S(a*(\overline{\theta}), b_s, \overline{\theta}) - [C + K] \tag{23}$$

for all $a < \underline{a}(\overline{\theta})$, *where* $\underline{a}(\overline{\theta})$ *is defined by* $v(\underline{a}(\overline{\theta}), b_0, \overline{\theta}) = C$ *and if*

$$\left[\frac{I - \underline{v}}{p + q} + \underline{v}\right]\left[\frac{v(a*(\overline{\theta}), b_s, \overline{\theta}) - v(a*(\overline{\theta}), b_0, \overline{\theta})}{v(a*(\overline{\theta}), b_0, \overline{\theta})}\right] \geq b_s. \tag{24}$$

Furthermore, expected payoffs from this contract are given by

$$U^E = ES(a*(\theta), b*(\theta), \theta) - (p + q)K \tag{25}$$

$$U^{VC} = (p + q)K. \tag{26}$$

To see how the convertible security works, let us consider the three states of the world in turn. In the bad state, E knows that she cannot repay $C+K$, so she chooses $a = 0$ (which is efficient) and VC gets all the returns of the project. Thus, VC is residual claimant on profits and has the right incentives to choose b_c.

In the medium state of the world, $C+K$ has been chosen such that (a) it is not worth VC's while to exercise his conversion option, and (b) E can repay $C+K$ if she invested $a*(\theta_m)$. Thus, in this state, E is residual claimant on profits, so she will choose $a*(\theta_m)$, while VC gets a fixed payment and chooses the least cost action $b = b_0$, which is again efficient.

Note that in the bad and medium states of the world, the convertible security reduces to a debt contract. In the good state, however, a debt contract would work very poorly because it does not induce VC to get involved in the company. To induce VC to choose $b = b_s$, it is necessary that he holds some equity. This is achieved by giving VC the option to convert C into fraction $\alpha < 1$ of the equity of the firm, where C and α have been chosen such that the conversion option is profitable if and only if E invested at least $a*(\overline{\theta})$ and that α is sufficiently large to make it worth VC's while to invest $b = b_s$. This is the incentive mechanism described by Proposition 1. The main differences are that VC's action space is now discrete, which allows us to reduce α below 1 without distorting VC's investment incentives, and that E is protected by limited liability and may default on her debt.

Suppose that in the good state E chooses an investment level $a < \underline{a}(\overline{\theta})$, that is, $v(a, b_0, \overline{\theta}) < C + K$. In this case, E is no longer able to repay her debt, so VC's threatpoint payoff in the renegotiation game is reduced below $C+K$. Renegotiation will ensure that VC gets to own enough of the venture that it is worth his while to choose the efficient investment b_s. However, E gets fraction λ of the surplus from renegotiation. Thus, if this share of the surplus from renegotiation was larger than what E would get if she invested efficiently, E would be induced to underinvest. Condition (23) is necessary to rule this possibility out.

Proposition 3 also requires that (24) holds. Using (20) and (21), condition (24) is equivalent to

$$C \geq \alpha v(a*(\overline{\theta}), b_0, \overline{\theta}). \tag{27}$$

If this condition does not hold, then VC prefers to convert his debt and *not* to invest b_s in the good state even if E has chosen the efficient effort level $a*(\overline{\theta})$. Hence, if this condition was violated, the convertible security would induce VC not to invest in the good state.

C. Empirical Implications

C.1. Why Are Convertible Securities Rarely Used by Passive Outside Investors?

Proposition 3 requires conditions (24) and (23) to hold. These conditions offer an explanation for why convertible securities are frequently used in venture capital finance but very rarely by banks or other outside investors who get far less involved in the projects they finance. To see this, note that condition (24) requires that—holding the left term in brackets constant—the value added by VC choosing b_s rather than the least cost action b_0 has to be sufficiently large as compared to the cost of this investment. Thus, a convertible security induces efficient investments only if the contribution of VC in the good state is very important. If the outside investor does not contribute sufficiently to the prospects of the project, then a convertible security does not induce efficient investments. This is consistent with the empirical observation that convertible securities are prevalent in venture capital finance, where the active involvement of the venture capitalist is of crucial importance, while it plays only a minor role in the financing that is provided by banks or other passive outside investors whose involvement in the firms they finance is far less important.

Holding the right term in brackets constant, condition (24) is more likely to be satisfied if $\underline{v}$ and $p+q$ are small, that is, it requires that, for a given investment cost, the liquidation value in the bad state has to be sufficiently small and/or the probability of the bad state has to be sufficiently high. Again, this seems to be typical for projects financed by venture capital, that fail with a high probability and where the liquidation value often tends to zero. On the other hand, projects that are financed by banks are typically less risky and offer a much higher liquidation value that can be collateralized.

Consider now condition (23). It can be shown that this condition is implied by the stronger condition[14]

$$K \leq (1-\lambda)[S(a*(\theta), b_s, \overline{\theta}) - \underline{v}] - \frac{I - \underline{v}}{p+q}. \tag{28}$$

This condition requires that λ is not too large and/or that the maximum surplus generated in the good state is large enough as compared to the (risk-adjusted) investment cost I. Thus, again, the proposition shows why convertible securities work well for projects where returns in the good state of the world are very high

[14] See the end of the proof of Proposition 3 in the Appendix.

(which is often the case for venture-capital-backed companies), but not for other investment projects with less potential in the good state.

C.2. The Exercise Date of the Conversion Option and Automatic Conversion

The convertible security considered in Proposition 3 sets the exercise date of the conversion option at date 3, that is, after VC has completed his investment. This is actually very important. To see this, suppose that VC has the right to convert his debt into equity at some date 1.5, that is, after VC observed E's investment and the realization of the state of the world, but before VC has made his own investment. Suppose further that the good state materialized and that E did invest efficiently at date 1. In this case, VC has an incentive not to exercise his option but to claim $C+K$. For if he does not own any equity of the firm he has no incentive to invest, and without VC's investment, the firm is worth very little to E. Hence, E wants to renegotiate the ownership structure and to sell some fraction of her firm to VC in order to make it worth VC's while to invest b_s. However, if VC gets fraction $1-\lambda>0$ of the surplus from renegotiation, then E's payoff from investing $a*(\overline{\theta})$ is reduced, which induces her to invest less than $a*(\overline{\theta})$.[15]

The cause of the problem is that by insisting on being paid back his credit at date 1.5, VC can credibly threaten *not* to invest if E does not give him some additional share of the surplus.

The contract of Proposition 3 solves this problem by giving VC the right to exercise his option at date 3, that is, definitely after he has to make his investment decision. In this case, VC cannot credibly threaten not to invest if E invested efficiently, because E can simply refuse to renegotiate. If she does so, the old contract remains in place, and when it comes to date 2, it is optimal for VC to invest and, given that he invested, he cannot benefit at date 3 from not exercising his conversion option. Note that the exercise date of the option need not be a fixed date. The contract could simply say that VC has the right to convert at any time up to the time of an IPO. The only important thing is that VC cannot credibly threaten not to invest by letting his option expire.

There exists a second way to avoid this hold-up problem if there is some verifiable signal on E's performance that is verifiable after VC has made his investment. In this case, an automatic conversion clause can be used. Automatic conversion forces VC to convert C if the performance measure is above some critical value. If this critical value is chosen such that it corresponds to the first best effort level of E in the good state, then automatic conversion overcomes the hold-up problem that arises with the possibility of renegotiation and implements the first best.

There is strong empirical evidence showing that the conversion date of convertible securities is indeed put at the end of the relationship between E and VC. Gompers (1997, p. 16) reports that in his sample of 50 convertible preferred equity venture investments, 92 percent had mandatory conversion that occurs at the time of the IPO (which can be interpreted as date 3 in our model). Furthermore, a "reasonable estimate of the number of contracts that are converted is between

[15] For a formal analysis of this case see Nöldeke and Schmidt (1998).

twenty and thirty percent, the fraction of venture-backed projects that eventually go public. A small number are likely converted when a venture-financed company is acquired by an already public firm." This suggests that the conversion option is exercised for the very successful ventures only, as suggested by our model. Furthermore, Sahlmann (1990), Gompers (1997), and Kaplan and Strömberg (2000) report that a significant fraction of convertible securities used in venture capital finance include automatic conversion clauses that are contingent on some milestones to be achieved by E. Again, this is consistent with our model.[16]

C.3. The Fraction of Cash Flows Accruing to VC as a Function of Performance

Kaplan and Strömberg (2000) report that the fraction of total cash flow that goes to the venture capitalist is decreasing with the performance of the firm. This is an immediate implication of the use of convertible securities and consistent with Proposition 3. In equilibrium, VC gets all of the cash flow in the bad state, a fixed debt payment (or dividend) in the medium state, and a constant fraction of the cash flow in the good state. Hence, the fraction of cash flows accruing to VC is indeed decreasing in v.

III. Extensions and Robustness

A. Syndication of Venture Capital Investments

Lerner (1994) reports that syndication is frequently observed in the venture capital industry. Typically, there is one lead investor, but there may be several additional venture capitalists who participate in the financing of the project. Gorman and Sahlman (1989, p. 235) report that a venture capitalist acting as lead investor will invest 10 times the direct hours he would if he was not the lead investor. Thus, it is the lead investor who actively supports and monitors the entrepreneur, while the other investors play a more passive role.

How does the existence of syndication affect our results? Suppose that the additional investors hold convertible securities or debt claims (but no common stock). In this case, they do not affect the optimal effort incentives given to the entrepreneur and the lead investor if the conditions of Proposition 3 are met. In

[16] However, there may be additional or alternative reasons for why many contracts have mandatory conversion at the time of the IPO. For example, a complicated capital structure with convertible securities outstanding can make the IPO less valuable and/or more difficult to evaluate by public outside investors. Furthermore, automatic conversion could be beneficial in order to avoid hold-up problems that may arise because E is liquidity constrained and may not be able to repay the debt even though all milestones have been reached. I am grateful to a referee for pointing this out.

The only other formal paper that I am aware of that offers an explanation for the use of automatic conversion clauses is Cestone and Wright (2000). In their model, the venture capitalist has to be forced to convert debt into equity in order to prevent him from investing in a rival firm that would reduce the profits of the incumbent (and therefore the returns of the equity share of VC). However, in their model, the automatic conversion clause has to be conditional on the success of the rival firm, which does not seem to be the case empirically.

fact, syndication may be beneficial. Proposition 3 requires that λ, the share of the surplus from renegotiation that goes to E, is not too large. One possibility to weaken E's bargaining position is to have multiple investors, all of whom have to agree to the renegotiation outcome. For example, if we assume that the surplus from renegotiation is split according to the generalized Nash bargaining solution, then the share that accrues to E goes down with every additional investor. Hence, syndication can be used as a remedy to the hold-up problem in renegotiation.[17]

B. Stage Financing

Another typical characteristic of the venture capital industry is stage financing (see, e.g., Sahlman (1989, pp. 503–507)). The venture capitalist makes a limited financial commitment at the beginning. If the firm turns out to be successful and if certain milestones are reached, there is a second financing round in which the VC and/or another venture capitalist puts in an additional investment, and so on.

To see the effects of stage financing on the incentives provided by a convertible security, consider a two-period version of our model. The first period ends after date 2. If the bad or the medium state materialized in the first period, there is no further investment and the game ends as in Section II. However, if the good state of the world materialized, then there is a second financing round at the beginning of period 2, where a new investment I_2 can be committed to the project. Then, as in period 1, a new state of the world, θ_2, materializes, E has to decide on her effort level a_2, and VC decides on his action b_2. The second investment could be carried out by the same or by a different venture capitalist (which is not uncommon in the industry), but this does not affect the following argument. Let us call the first venture capitalist VC1 and the second VC2. Now we can again employ a convertible security to induce both, E and VC2, to invest efficiently in period 2. Given the contract of the first period, this second contract gives rise to certain expected payoffs for E and VC1. Given that both, E and VC1, are assumed to be risk neutral, this expected payoff can simply be substituted in the payoff functions of Section II without affecting the incentives given to the two parties. Thus, they will still invest efficiently in period 1.[18]

C. Multiple Securities

VCs often hold multiple claims on the firms they finance. This could affect investment incentives. To see this, recall that in the medium state of the world, E is induced to spend the efficient level of effort because she is full residual claimant on profits. This requires that VC does not hold, in addition to his convertible

[17] A similar argument is made in different contexts by Bolton and Scharfstein (1996) and Lülfesmann (2001). However, it should be noted that there are also several other potential explanations for the widespread use of syndication. See Lerner (1994) for a discussion.

[18] Note, however, that it is important that there is a second financing round only in the good state. If there was a second round in the medium state as well, E would no longer be full residual claimant on profits in the first period in this state, so she would underinvest.

security, other claims that make him participate in an increase in profits in this state of the world. In particular, VC must no hold common stock.[19]

What types of securities do VCs hold? Kaplan and Strömberg (2000, Panel G of Table 1, p. 50) report that 167 out of 200 financing rounds in their sample used convertible securities only. An additional 6 used convertible preferred stock and straight preferred stock. Hence, in more than 85 percent of their sample, VCs do not hold additional claims that may affect investment incentives.[20]

D. Multidimensional Effort

So far we assumed that E's investment is one-dimensional, $a \in \mathbb{R}_0^+$. However, the entrepreneur has to engage in many different tasks in order to make the venture a success. So suppose that E has to choose a multidimensional effort vector, $a \in \mathscr{A} \subset \mathbb{R}^N$. For example, E may have to invest in R&D, she may have to spend effort in order to hire and motivate employees, she may have to invest in marketing, in supplier, and customer relations, and so forth. Assume that for each state of the world, there is a well-defined optimal investment vector $a^*(\theta)$. Then the convertible debt contract of Proposition 3 still implements the first best investments of both parties. In the bad state of the world, VC is going to take over and to liquidate the firm or to replace the entrepreneur, so it is still optimal and efficient for E not to invest at all. In the medium state, E is full residual claimant on profits, so she will invest efficiently, no matter whether a is one- or multidimensional. In the good state, she wants to choose a so as to make it worth VC's while to exercise his conversion option. Recall that if VC exercises his option, then E gets the entire social surplus minus a constant. Hence, E has an incentive to increase the value of the venture in the most efficient way, because private and social incentives to invest coincide, so E is going to invest efficiently even if her effort is multidimensional.

E. The Timing of Information Flows and Additional Uncertainty

It could be argued that it would be more realistic to assume that the information on the realization of the state of the world is not revealed before E has made her investment decision. If E does not know the realization of θ before she invests, she cannot choose the ex post efficient effort level $a^*(\theta)$. Instead, she would have to choose an ex ante optimal effort level a^* (unconditional on θ) that maximizes *expected* social surplus. It turns out that, in this case, a convertible security can also

[19] I am grateful to the editor for pointing out this important observation.

[20] However, there are some so-called participating convertible preferreds in this sample. Upon liquidation or exit of a participating convertible preferred, investors receive both the principal amount of the preferred and they receive common stocks. From the data provided by Kaplan and Strömberg, it is impossible to compute the exact number of the 173 financing rounds mentioned above that used a participating convertible preferred. But the total of participating preferreds was 72, so it must be the case that in more than 100 financing rounds, venture capitalists used regular convertible securities only. See Hellmann (2000) for an interesting explanation of why participating convertible preferred securities might be used if there is a conflict of interest about the exit channel between E and VC.

be used to implement the first best. In fact, this case is somewhat simpler than the case considered in Section II, because the investment incentives for E do not have to be optimal in each state, but only on average. In a previous version of this paper (Schmidt 1999), it is shown formally how to construct a convertible security that induces first best investment incentives for the case with and without renegotiation.

As a further extension of the model, suppose that there is some uncertainty in addition to the realization of θ. Any uncertainty that resolves after VC invested and exercised his conversion right is no problem. We can simply interpret $v(a, b, \theta)$ as an expected value. Given that both parties are risk neutral, this is all they care about.

If there is additional uncertainty that resolves after the initial investment I has been sunk but before E invests, the parties could renegotiate the initial contract and adapt the conversion rate without distorting the investment incentives as is shown in Proposition 2. Given that both parties are symmetrically informed, they will always reach an efficient agreement.

Finally, there could be some additional uncertainty that resolves after the realization of θ and after E invested but before VC made his investment decision. Again, if the parties are free to renegotiate, they will always adapt the conversion rate such that VC is induced to invest efficiently. It can be shown that if the uncertainty is not too large, then the maximum remains at a^* and E is still going to invest efficiently.[21]

F. The Timing of Investments

The assumption that E and VC invest sequentially is important for our first best results to hold. In the context of venture capital finance, this assumption is not unrealistic. The entrepreneur often has the strongest impact on her company at the beginning of the project. On the other hand, the active involvement of the venture capitalist may be more important at the expansion stage, for example, when key employees have to be attracted and strategic business partners have to be found, or when things turn sour and VC has to step in to replace the entrepreneur and/or liquidate the company.

However, in reality, these investments are not taken at a single and fixed point in time but rather over an extended period of time. Furthermore, as was shown in the discussion of the exercise date of the conversion option after Proposition 3, VC may have a strategic incentive to delay his action in order to force renegotiation and to get a larger share of the surplus. Our results still hold if the time intervals in which E and VC have to choose their investments overlap, and if the parties can costlessly delay their investments within these intervals. However, to implement the first best, it is important that VC can complete his investment after he observed E's total investment and that he must complete his investment before date 3, the exercise date of the option.

[21] See Nöldeke and Schmidt (1998, Sec. 5) for a formal analysis of this case.

G. First Best versus Second Best

The preceding sections have shown that our results are robust to many variations of the model, but they are clearly not robust to all changes. For example, if E and VC invest simultaneously, a convertible security cannot implement first best investment incentives. Also, if there is discounting and if VC can strategically delay his investment, there may be a hold-up problem that cannot be fully resolved by a convertible security.[22] However, it is important to note that the incentive mechanism provided by a convertible security, which induces E to work harder in order to make it worth VC's while to exercise his conversion option and to induce VC to get involved in the project, can be very useful in a second best world, too. In fact, it is easy to see that a contract based on convertible securities must always at least weakly outperform a standard debt–equity contract. The reason is that a convertible debt contract contains both debt and equity as special cases. If the conversion ratio is very favorable to VC, he will always use his conversion option and a convertible security is equivalent to equity. If the conversion ratio is very unfavorable, then VC will never exercise his option, and a convertible security is equivalent to debt. Hence, any simple debt–equity contract can be replicated by a contract that is based on an appropriately chosen convertible security.

IV. Conclusions

Convertible securities endogenously allocate cash flow rights as a function of the realization of the state of the world and of the effort provided by the entrepreneur. By choosing the conversion rate appropriately, the venture capitalist is going to invest efficiently if and only if the entrepreneur invested at least the efficient amount. This generates a kink in the entrepreneur's payoff function that induces her to invest efficiently. Thus, convertible securities are a powerful incentive mechanism that can induce efficient investments without making both parties residual claimant on the margin.

We have specialized the model to venture capital finance by assuming that the entrepreneur is wealth constrained and protected by limited liability and that the venture capitalist's action is most important in the bad state, when the first has to be liquidated, and in the good state, if the venture is a high flier, but not if the project is mediocre. The optimal contract in this setup closely resembles the convertible securities that are frequently observed in venture capital finance. Furthermore, the model can explain why convertibles are not being used by banks or outside investors who do not get involved in the management of the firms they finance.

Our model is consistent with many other empirical findings on venture capital. It can explain the frequent use of automatic conversion clauses and why the conversion date is often set at the date when VC exits. It is consistent with Kaplan and Strömberg's (2000) finding that the fraction of cash flow that goes to VC is decreasing with the success of the venture, and it is consistent with the wide-

[22] I am grateful to a referee for this observation.

spread use of stage financing and of syndication of venture capital investments. Furthermore, the results are robust to changes in the time structure of information flows and investments. However, the sequential nature of investments is important to implement first best efficient investments. If investments are simultaneous, the first best cannot be implemented by convertible securities, but they still dominate standard debt–equity contracts.

Appendix

Proof of Proposition 1: Suppose that E has chosen $a^*(\theta)$. If VC exercises his conversion option, his payoff is

$$U^{VC} = v(a^*(\theta), b, \theta) - b - F - I. \tag{A1}$$

Hence, VC is full residual claimant of total social surplus minus a constant, so he will invest efficiently and choose $b = b^*(\theta)$, which yields a payoff (after substituting F) of

$$U^{VC} = v(a^*(\theta), b^*(\theta), \theta) - b^*(\theta) - v(a^*(\theta), b^*(\theta), \theta) + b^*(\theta) + I - K - I = K. \tag{A2}$$

If VC does not exercise his conversion option, he gets a fixed payment C, so he will invest $b = 0$ and his payoff is

$$U^{VC} = C - I = K. \tag{A3}$$

Thus, if E has chosen $a^*(\theta)$, VC is indifferent between investing efficiently (and exercising his conversion option) and not investing at all (and not converting). Note that in equilibrium, VC must invest $b^*(\theta)$ and exercise his option. Otherwise E would have invested slightly more than $a^*(\theta)$ in order to break his indifference, but, by a standard "shaving argument," this cannot be an equilibrium.

Consider now E's optimal investment strategy. Note that $v(a, b, \theta)$ is strictly increasing in a. Thus, if E chooses $a > a^*(\theta)$, it is strictly optimal for VC to exercise his conversion option. Therefore E will lose all the equity of his firm and receive the fixed payment F. Hence, investing strictly more than $a^*(\theta)$ cannot be optimal. So suppose that E chooses $a < a^*(\theta)$. Note that VC can still guarantee himself a payoff of K by choosing $b = 0$ and insisting to be repaid C. Of course, this would be inefficient and the two parties would now renegotiate the initial contract in order to make sure that VC becomes full residual claimant on the margin and invests efficiently. However, total social surplus is smaller than with $a = a^*(\theta)$, and VC will not agree to a new contract that gives him less than his outside option of K. Therefore, E's payoff after renegotiation must be smaller than

$$S(a^*(\theta), b^*(\theta), \theta) - I - K \tag{A4}$$

which is what she would have received if she had chosen $a = a^*(\theta)$. Q.E.D.

Proof of Proposition 2: Without renegotiation VC would get the fixed payment C and thus choose $b = 0$ at date 2, independent of E's investment. Thus, without renegotiation, E would choose $\hat{a}(\theta)$ as defined by (13), so the threatpoint of renegotiation is given by

$$\hat{U}^E = v(\hat{a}(\theta), 0, \theta) - \hat{a}(\theta)C \quad \text{(A5)}$$

$$\hat{U}^{VC} = C - I \quad \text{(A6)}$$

If the parties renegotiate the contract by giving VC the option to convert his debt claim into 100 percent of the equity of the firm by paying the additional amount

$$F = v(a*(\theta), b*(\theta), \theta) - b*(\theta) - C \quad \text{(A7)}$$

then, by Proposition 1, both parties would be induced to invest efficiently and total social surplus would be maximized. The surplus of renegotiation

$$S(a*(\theta), b*(\theta), \theta) - v(\hat{a}(\theta), 0, \theta) + \hat{a}(\theta) \quad \text{(A8)}$$

will be split in proportions $\lambda, 1-\lambda$ between E and VC, so payoffs after renegotiation are given by

$$U^E = v(\hat{a}(\theta), 0, \theta) - \hat{a}(\theta) - C + \lambda[S(a*(\theta), b*(\theta), \theta) - v(\hat{a}(\theta), 0, \theta) + \hat{a}(\theta)] \quad \text{(A9)}$$

$$U^{VC} = C - I + (1-\lambda)[S(a*(\theta), b*(\theta), \theta) - v(\hat{a}(\theta), 0, \theta) + \hat{a}(\theta)] \quad \text{(A10)}$$

Thus, expected payoffs as of date 0 are given by

$$EU^E = \int_\theta \lambda S(a*(\theta), b*(\theta), \theta) + (1-\lambda)[v(\hat{a}(\theta), 0, \theta) - \hat{a}]\mathrm{d}G(\theta) - C \quad \text{(A11)}$$

$$EU^{VC} = C - I + (1-\lambda)\int_\theta [S(a*(\theta), b*(\theta), \theta) - v(\hat{a}(\theta), 0, \theta) + \hat{a}(\theta)]\mathrm{d}G(\theta) \quad \text{(A12)}$$

Substituting (14), we get

$$EU^E = \int_\theta S(a*(\theta), b*(\theta), \theta) - I - K \quad \text{(A13)}$$

$$EU^{VC} = K \quad \text{(A14)}$$

Q.E.D.

Proof of Proposition 3: Note first that if we substitute (20) in (21), then the inequality part of (21) is equivalent to $v(a*(\overline{\theta}), b_s, \overline{\theta}) - b_s > \frac{I-\underline{v}}{p+q} + \underline{v}$. This inequality must be satisfied because

$$v(a^*(\overline{\theta}), b_s, \overline{\theta}) - b_s > v(a^*(\overline{\theta}), b_0, \overline{\theta}) > v(a^*(\theta_m), b_0, \theta_m) > v(a^*(\theta_m), b_0, \theta_m) - a^*(\theta_m) > \frac{I - \underline{v}}{p+q} + \underline{v}, \tag{A15}$$

where the last inequality follows from (19). Hence, it is indeed the case that $\hat{\alpha} < 1$.

Consider now the three states of the world in turn:

(a) State $\underline{\theta}$: Clearly, $C + K = \frac{I-\underline{v}}{p+q} + \underline{v} + K > \underline{v}$. Hence, the maximum total surplus in this state is strictly smaller than $C+K$, so E is unable to repay her debt in this state. Hence, by not exercising his conversion option, VC will get all the returns of the project. Therefore, it is optimal for VC to choose $b = b_c$, which is efficient in this state and guarantees him a payoff of $\underline{v} > v(a, b, \underline{\theta}) - b$ for all $a \in R_0^+$ and $b \in \{b_0, b_s\}$. Anticipating that she cannot repay $C+K$, E knows that there is no return to her investment, so she will choose $a^*(\underline{\theta}) = 0$, which is also efficient. Note that there is no scope for renegotiation in this state of the world.

(b) State θ_m: Define $\underline{a}(\theta_m)$ by $v(\underline{a}(\theta_m), b_0, \theta_m) = C + K$. Conditions (19) and (22) imply that $\underline{a}(\theta_m)$ exists and is strictly smaller than $a^*(\theta_m)$. We have to distinguish three cases:

(i) Suppose that E has chosen $a < \underline{a}(\theta_m)$. In this case, if VC chooses $b = b_0$ at date 2, then he should not exercise his conversion option, because E will be unable to repay her debt, so the entire return goes to VC and he receives $v(a, b_0, \theta_m) - I$. By Assumption 1(b), this is larger than the maximum return that he could hope for if he chooses $b = b_s$ or $b = b_c$. Hence, in this case it is optimal for VC to choose $b = b_0$ and not to convert.

(ii) Now suppose that $\underline{a}(\theta_m) \le a \le a^*(\theta_m)$. Again, choosing $b = b_c$ is dominated by $b = b_0$ and not exercising the conversion option. If VC chooses $b = b_0$ and converts his debt, he gets

$$\begin{aligned}\alpha v(a, b_0, \theta_m) + K - I &= (C + b_s)\,\frac{v(a, b_0, \theta_m)}{v(a^*(\bar{\theta}), b_s, \bar{\theta})} + K - I \\ &< \left(C + C\,\frac{v(a^*(\bar{\theta}), b_s, \bar{\theta}) - v(a^*(\bar{\theta}), b_0, \bar{\theta})}{v(a^*(\bar{\theta}), b_0, \bar{\theta})}\right) \\ &\quad \frac{v(a, b_0, \theta_m)}{v(a^*(\bar{\theta}), b_s, \bar{\theta})} + K - I \\ &= C\,\frac{v(a, b_0, \theta_m)}{v(a^*(\bar{\theta}), b_0, \bar{\theta})} + K - I \le C + K - I,\end{aligned} \tag{A16}$$

where (21) and (24) have been used. Hence, this cannot be better than choosing $b = b_0$ and not converting. Nor can it be better to invest $b = b_s$ and not to exercise the conversion option, which yields $C+K-$

$b_s - \mathrm{I} < C + K - I$. Finally, VC cannot improve his payoff by investing $b = b_s$ and exercising his conversion option, because

$$\begin{aligned} \alpha v(a, b_s, \theta_m) - b_s + K - I &< \alpha[v(a, b_s, \theta_m) - b_s] + K - I \\ &< \alpha v(a, b_0, \theta_m) + K - I \; < \; C + K - I \end{aligned} \tag{A17}$$

where the second inequality is implied by Assumption 1(b) and the last inequality follows from the argument given in (A16). Hence, we have shown that for all a with $\underline{a}(\theta_m) \leq a \leq a*(\theta_m)$, VC will not invest and not exercise his conversion option in state θ_m.

(iii) Finally, if $a > a^*(\theta_m)$, then VC must get at least a payoff of $C+K-I$, which he can guarantee himself by not converting and choosing $b = b_0$. We will show below that this is sufficient to induce E to always choose $a \leq a^*(\theta_m)$, so we do not have to analyze this case further.

Consider now the optimal choice of a for E in state θ_m. If E chooses $a < \underline{a}(\theta_m)$, then her payoff is $-a$ and she should choose $a = 0$, which guarantees a payoff of 0. If she chooses a such that $\underline{a}(\theta_m) \leq a \leq a*(\theta_m)$, then she can repay her debt and, since VC is not going to exercise his conversion option, she is residual claimant on profits at the margin. Hence, in this range, the optimal choice of a is $a^*(\theta_m)$, which maximizes total surplus in state θ_m and gives a payoff strictly larger than 0 to E by (19). Thus, $a = a^*(\theta_m)$ is better than $a = 0$. Finally, it cannot be optimal to choose $a > a^*(\theta_m)$. This reduces social surplus (by the definition of $a^*(\theta_m)$), while VC still gets at least $C+K-I$, so E's payoff must go down. Hence, we have shown that in state θ_m, E's payoff is maximized by investing the efficient amount $a^*(\theta_m)$, and VC does not invest and he does not exercise his conversion right in this state in equilibrium.

Note that renegotiation is not an issue in the medium state either. VC will choose the least cost action b_0 (which is efficient) for all $a \in R_0^+$ and no matter whether he converts C or not, and E is induced to choose $a^*(\theta_m)$. Hence, there is no scope for an efficiency improvement through renegotiation.

(c) State $\overline{\theta}$: First, we want to show that it is optimal for VC to invest $b = b_s$ and to exercise his option if and only if $a \geq a*(\overline{\theta})$. Suppose that E invested at least the efficient amount, that is, $a \geq a*(\overline{\theta})$. Note that VC can guarantee himself a payoff of $C+K-I$ by choosing $b = b_0$ and not converting C. Clearly, it cannot be optimal to choose $b = b_c$. Nor can it be optimal to choose $b = b_s$ and not to exercise the option because this would yield $C+K-b_s-I < C+K-I$. On the other hand, if VC does invest and does exercise his option, then his payoff is given by

$$\begin{aligned} U^{VC}(a) &= \alpha v(a, b_s, \overline{\theta}) - b_s + K - I \\ &= \frac{C + b_s}{v(a*(\overline{\theta}), b_s, \overline{\theta})} v(a, b_s, \overline{\theta}) - b_s + K - I. \end{aligned} \tag{A18}$$

Note that $U^{VC}(a)$ is monotonically increasing in a. VC's payoff if E invested $a*(\overline{\theta})$ is

$$U^{VC}(a*(\overline{\theta})) = \frac{C + b_s}{v(a*(\overline{\theta}), b_s, \overline{\theta})} v(a*(\overline{\theta}), b_s, \overline{\theta}) - b_s + K - I$$
$$= C + K - I. \tag{A19}$$

Hence, VC prefers to invest b_s and to convert C rather than not to invest ($b = b_0$) and not to convert if E invested $a \geq a*(\overline{\theta})$.

To complete this step of the argument it has to be shown that, given $a \geq a*(\overline{\theta})$, VC prefers to invest b_s and to convert C rather than not to invest and to convert C. Hence, we have to show that

$$\alpha v(a, b_s, \overline{\theta}) - b_s + K - I \;\geq\; \alpha v(a, b_0, \overline{\theta}) + K - I \tag{A20}$$

for all $a \geq a*(\overline{\theta})$. Substituting (21) for α this is equivalent to

$$\frac{C + b_s - b_0}{v(a*(\overline{\theta}), b_s, \overline{\theta})} \left[v(a, b_s, \overline{\theta}) - v(a, b_0, \overline{\theta}) \right] \geq b_s \tag{A21}$$

Note that by Assumption 1(d) the left hand side of (A21) is monotonically increasing with a. Hence, if (A21) holds for $a = a*(\overline{\theta})$, it also holds for all $a > a*(\overline{\theta})$. Substituting $a = a*(\overline{\theta})$, (20) for C, and rearranging yields (24). Hence, we have shown that it is optimal for VC to invest and to convert his debt in state $\overline{\theta}$ if $a \geq a*(\overline{\theta})$; so in this range of a, he is induced to invest efficiently and there is no scope for renegotiation.

Consider now E's investment if she is restricted to choosing $a \geq a*(\overline{\theta})$. If she chooses $a = a*(\overline{\theta})$, VC will invest b_s and exercise his conversion option, so E's payoff is

$$U^E(a*(\overline{\theta}), \overline{\theta}) = (1 - \alpha) v(a*(\overline{\theta}), b_s, \overline{\theta}) - a*(\overline{\theta}) - K \tag{A22}$$

Recall that by (21) $\alpha v(a*(\overline{\theta}), b_s, \overline{\theta}) = C + b_s$. Substituting this in (A22) we get

$$\begin{aligned} U^E(a*(\overline{\theta}), \overline{\theta}) &= v(a*(\overline{\theta}), b_s, \overline{\theta}) - b_s - a*(\overline{\theta}) - C - K \\ &= S(a*(\overline{\theta}), b*(\overline{\theta}), \overline{\theta}) - [C + K] \end{aligned} \tag{A23}$$

which is weakly positive by (22). Hence, if E chooses $a = a*(\overline{\theta})$, she gets the entire social surplus at $a = a*(\overline{\theta})$ minus the constant $C+K$. Investing more than $a*(\overline{\theta})$ reduces social surplus by the definition of the first best and increases the payoff of VC, who can always guarantee himself at least $C+K-I$. Thus, because E's and VC's payoffs add up to the social surplus, E's payoff must fall. Hence, E will never invest more than $a*(\overline{\theta})$.

Suppose now that E chooses $a < a*(\overline{\theta})$. Without renegotiation, VC will not exercise his conversion option and not invest b_s, which is inefficient. In this case, there is scope for an efficiency improvement, so the parties will renegotiate the initial contract in order to induce VC to convert C and to invest b_s. Define $\underline{a}(\overline{\theta})$ by $v(\underline{a}(\overline{\theta}), b_0, \overline{\theta}) = C + K$. Consider first a "small" deviation from $a*(\overline{\theta})$ such that $\underline{a}(\overline{\theta}) \leq a < a*(\overline{\theta})$. In this case, E is able to repay her debt, so VC can still guarantee himself a return of $C+K-I$ by choosing b_0 and not converting C. Clearly, VC will not agree to a new contract that gives him less than this. E's payoff after renegotiation is given by

$$\begin{aligned} U^E(a \mid \overline{\theta}) =& v(a, b_0, \overline{\theta}) - C - K - a + \lambda\big[v(a, b_s, \overline{\theta}) - b_s - v(a, b_0, \overline{\theta})\big] \\ =& \lambda[v(a, b_s, \overline{\theta}) - b_s] + (1-\lambda)v(a, b_0, \overline{\theta}) - [C+K] - a \\ <& v(a, b_s, \overline{\theta}) - b_s - a - [C+K] \\ <& v(a*(\overline{\theta}), b_s, \overline{\theta}) - b_s - a*(\overline{\theta}) - [C+K] \;=\; U^E(a*(\overline{\theta}) \mid \overline{\theta}). \end{aligned} \tag{A24}$$

The last inequality follows from the definition of the first best. The intuition is simply that if E invests (slightly) less than $a*(\overline{\theta})$, then the total surplus becomes smaller while the return to VC does not decrease. Hence, such a deviation cannot pay for her.

Consider now a "large" deviation, that is, $a < \underline{a}(\overline{\theta})$. In this case, E cannot repay her debt. Thus, E's expected payoff after renegotiation is given by

$$U^E(a < \underline{a}(\overline{\theta}) \mid \overline{\theta}) \;=\; \lambda\big[v(a, b_s, \overline{\theta}) - b_s - v(a, b_0, \overline{\theta})\big] - a \tag{A25}$$

If E chooses $a = a*(\overline{\theta})$, her payoff is given by

$$U^E(a*(\overline{\theta})|\overline{\theta}) \;=\; v(a*(\overline{\theta}), b_s, \overline{\theta}) - b_s - a*(\overline{\theta}) - [C+K] \tag{A26}$$

Hence, if

$$\lambda\big[v(a, b_s, \overline{\theta}) - b_s - v(a, b_0, \overline{\theta})\big] - a \leq v(a*(\overline{\theta}), b_s, \overline{\theta}) - b_s - a*(\overline{\theta}) - [C+K] \tag{A27}$$

E prefers to choose $a = a*(\overline{\theta})$. This is equivalent to condition (23) in the proposition.

Note that

$$\begin{aligned} &\lambda\big[v(a, b_s, \overline{\theta}) - b_s - v(a, b_0, \overline{\theta})\big] - a \\ &< \lambda\big[v(a, b_s, \overline{\theta}) - b_s - a - v(a, b_0, \overline{\theta})\big] \\ &< \lambda\big[v(a*(\overline{\theta}), b_s, \overline{\theta}) - b_s - a*(\overline{\theta}) - v(a, b_0, \overline{\theta})\big] \\ &< \lambda[S(a*(\overline{\theta}), b_s, \overline{\theta}) - \underline{v}] \end{aligned} \tag{A28}$$

Hence, a sufficient condition for (23) to be satisfied is

$$\lambda[S(a*(\overline{\theta}), b_s, \overline{\theta}) - \underline{v}] \;\leq\; S(a*(\overline{\theta}), b_s, \overline{\theta}) - [C+K] \tag{A29}$$

Substituting (20) and rearranging gives

$$K \leq (1-\lambda)[S(a*(\theta), b_s, \overline{\theta}) - \underline{v}] - \frac{I - \underline{v}}{p+q} \tag{A30}$$

Finally, given the convertible debt contract, VC's expected equilibrium payoff is given by

$$U^{VC} = p[\alpha v(a*(\overline{\theta}), b_s, \overline{\theta}) + K - b_s] + q[C+K] + (1-p-q)\underline{v} - I$$
$$= (p+q)[C+K] + (1-p-q)\underline{v} - I \;=\; (p+q)K \tag{A31}$$

by the definition of C. E's expected payoff is given by

$$U^E = p\cdot[S(a*(\overline{\theta}), b_s, \overline{\theta}) - (C+K)] + q\cdot[S(a*(\theta_m), b_0, \theta_m) - (C+K)]$$
$$+ (1-p-q)\cdot 0 \tag{A32}$$
$$= ES(a*(\theta), b*(\theta), \theta) - (p+q)K.$$

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