Commitment in Games with Asymmetric Information

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Klaus M. Schmidt

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Contents

Chapter 1:

Repeated Games, Incomplete Contracts and Commitment

1.1. Introduction

In his classical book on noncooperative game theory Schelling (1960) repeatedly stresses the importance of commitment in many social interactions. No matter whether the involved parties have to solve a coordination problem, whether they compete on a market, fight a war, or bargain on the division of a pie, if one of the players can credibly bind himself to a certain course of action this commitment has an important impact on the outcome of the game. For example,

> *"if the institutional environment makes it possible for a potential buyer to make a 'final' offer subject to extreme penalty in the event he should amend the offer - to commit himself - there remains but a single, well-determined decision for the seller: to sell at the price proposed or to forego the sale. The possibility of commitment converts an indeterminate bargaining situation in a two move game; one player assumes a commitment, and the other makes a final decision. The game has become determinate."*[1)]

In this example the possibility of commitment is given exogenously. To put it in the language of noncooperative game theory it is an element of the strategy set of one of the players. He moves first and is given the choice whether or not to commit himself.

This dissertation, however, focuses on the possibility of commitment which arises endogenously in equilibrium because one of the players has private information about some of his characteristics. While he cannot commit himself directly by

[1)] Schelling (1960, p. 121f)

taking an irreversible action he can use his private information to credibly threaten to behave as if he were committed to a certain course of action. In Chapter 2 and 3 it is shown how "commitment through incomplete information" works in repeated games in which "reputation" plays an important role. In these chapters the asymmetric information is taken as given and it is analysed how players learn and build up reputations in course of their relationship. In Chapter 4 the distribution of information is endogenously determined in an "incomplete contracts" model. There a situation is considered where players may deliberately choose the distribution of information through the choice of a "governance structure" in the beginning of the game, and it is shown that this gives rise to a commitment effect.

Before going into further details of the models this chapter gives an overview on the literatures on repeated games with asymmetric information and on incomplete contracts, and sets out the problems which are discussed in this dissertation.

1.2. Repeated Games and the "Reputation Effect"

A common intuition is that if players are engaged in a repeated relationship then experiences they have made in the past will have an important impact on their future play. Usually this is attributed to the fact that players care about "reputations". In common language "reputation" is a characteristic ascribed to one person by another, e.g. "A has a reputation for being tough". The characteristic itself ("toughness"), however, is not observable. So "reputation" is primarily an empirical statement on observed past behaviour ("A has been observed in the past to act as if he were tough"). The question is why past observations should have any predictive power for future behaviour.

To answer this question in game theoretic terms suppose player A is privately informed about some of his characteristics, e.g. his preferences or his strategy set, which will be called his “type”.[2] In a repeated game a strategy for player A prescribes which action to take in any period depending on the history of the game up to that period and on his private information. In equilibrium the other players have a supposition about what the strategy of each possible type of player A is. So they may infer some information about player A’s type from the observations of his past play, i.e. they may repeatedly use Bayes’ rule to update their probability assessment about his possible types. Since player A’s strategy depends on his private information these inferences can be used to improve predictions of his future play.[3]

Going one step further, if the other players use the observations of player A’s past play to learn about his type, player A will take the impact of his behaviour on his reputation (and thus on his future payoff) into account. He could try to manipulate the inferences of the other players through certain actions today in order to get a higher payoff in the future. But, of course, if the other players are fully rational they will foresee any such behaviour, so they cannot be “fooled” in equilibrium. Nevertheless it has been shown that there are games in which player A can use the fact that there is a small amount of incomplete information about his type to enforce an equilibrium outcome which is much more favourable to him than what he would have got under complete information. This has been called “reputation effect” and has found considerable attention in the literature.

Two classes of repeated games have to be distinguished. Either the same players play the same game over and over again, as, for example, a downstream and an

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[2] The notion of a player’s “type” (i.e. his payoff function or strategy set) goes back to Harsanyi (1967-68), who demonstrated how to complete a “game” with incomplete information to a game with imperfect information. Only the latter one is a well defined game.

[3] This discussion of “reputation” follows Wilson (1986). In particular we restrict the usage of the term “reputation” to games in which there is some private information about at least one of the players persistent over time. Otherwise there is nothing for which a reputation can be build up. However, some authors use the term “reputation” also in repeated games with complete information. Here “reputation” just means that a particular equilibrium is selected (see e.g. R. Barro and D. Gordon (1983)).

upstream firm who have to bargain on the terms of trade of an input good every period. We will refer to these games as “two long-run player games”. Or there is only one long-run player who plays the same game against a “sequence” of short-run opponents each of whom is interested in his one period payoff only but observes all previous play. This is, for example, the situation of a manufacturer who sells his products to a sequence of different customers, each of whom buys only once but can observe past prices and qualities.

For the latter class of games some quite strong results have been established. The first formalizations of a reputation effect in such a game are due to Kreps and Wilson (1982a) and Milgrom and Roberts (1982). They consider the chain-store game introduced by Selten (1978). Suppose there is a multi-market monopolist who faces a succession of potential entrants to each of his markets. If there is entry the monopolist may either “fight” (a price war) or “acquiesce” (to share the market and collude), where “fight” yields less profits than acquiescence. But, of course, the monopolist prefers most the situation where there is no entry at all and he can keep his monopoly. From the perspective of each entrant entry is profitable only if the monopolist acquiesces.

If there were only one market the monopolist would clearly acquiesce, and - anticipating this - the entrant would enter. But suppose that there are many markets. A common intuition is that the monopolist will fight in the beginning in order to deter future entry and that acquiescence may occur only in the last few periods of the game. However, the unique subgame perfect equilibrium of the finitely repeated game prescribes that the monopolist acquiesces in every period and every entrant enters.[4] The reasoning follows a simple backwards induction argument. The monopolist will acquiesce in the last period anyway since nothing can be gained in

[4] Note that there are many other equilibria if the game is infinetely repeated, in which case the Folk Theorem of Fudenberg, Kreps and Maskin (1990) applies.

the future if he fights. But then his play in the second last period has no influence on the outcome of the last period, so he should acquiesce in this period as well, and so on by induction. This is Selten's celebrated chain-store paradox. Kreps and Wilson (1982a) and Milgrom and Roberts (1982) demonstrate that if there is a small prior probability that the monopolist is a "tough" type who prefers fighting to acquiescence, then a "reputation effect" may overcome the unravelling from the end. There is a sequential equilibrium in which the weak monopolist behaves as if he were tough in all but the last few periods of the game. Since this behaviour is rationally expected by his opponents there will be no entry until close to the end.[5)] Thus, due to a small amount of incomplete information, the monopolist gets a much higher payoff in equilibrium than he would have got under complete information.

This result has been generalized and considerably strengthened by a recent paper of Fudenberg and Levine (1989). They consider the class of all simultaneous move games in which a long-run player faces a sequence of short-run opponents, each of whom is interested only in his one-period payoff but observes all previous play. Suppose that the information structure of the game is perturbed, such that with an arbitrarily small but positive probability player one is a "commitment type" who always plays the same "commitment strategy" in each period. Fudenberg and Levine show that if the discount factor of the long-run player goes to 1 then his equilibrium payoff in any Nash equilibrium is bounded below by his "commitment payoff", i.e. the payoff he would have got if he could have committed himself publicly always to take the commitment strategy.

This result is very strong for at least three reasons. First of all it gives a clearcut prediction of the equilibrium payoffs in all Nash equilibria and is not restricted to sequential equilibria or equilibria satisfying any other refinement. Second, it holds

[5)] There are other sequential equilibria with different equilibrium paths as well, but this is the unique "stable" equilibrium.

for all simultaneous move games and can be generalized (although with an important qualification) to extensive form games as well. In Fudenberg and Levine (1991) they can extend this result to games in which player one would like to commit himself to a mixed strategy and to games with moral hazard, in which only a noisy signal about player one's action is observable. So the result concerns a quite general class of games. Finally, it is robust against further perturbations of the players' payoff functions. No matter what other possible types of the long-run player exist with positive probability and how likely they are to occur the "commitment type" will dominate the play.

Fudenberg and Levine's theorem is based on two assumptions. The commitment type must have positive probability and player two must be completely myopic, i.e. she has to be interested only in her one period payoff. The first assumption seems to be rather mild, given that the prior probability of the commitment type may be arbitrarily small and that there may be all kind of other possible types around. However, the second assumption clearly is a severe restriction. Nevertheless, in this particular class of games, in which a long-run player faces a sequence of short-run opponents, the reputation effect has been shown to be very powerful.

Unfortunately, the literature on repeated games with two or more long-run players has been much less successful, although the beginnings were highly promising. The pioneering work was done by Kreps, Milgrom, Roberts and Wilson (1982) in their seminal work on the finitely repeated prisoner's dilemma. It is well known that the only Nash equilibrium in any finite repetition of the prisoner's dilemma game is for both players always to "defect". Kreps et.al. demonstrate that if the information structure of this game is perturbed such that with an arbitrarily small but positive probability one of the players is committed to the "tit-for-tat" strategy then the payoffs of all sequential equilibria are close to the "cooperative" outcome. Furthermore, Kreps et.al. introduced a new method, namely to characterize rather than to calculate the set of equilibria.

While the incomplete information approach to two long-run player games has been very popular in the beginning of the 1980s to solve all kind of puzzles in game theory, industrial organization, macroeconomics, etc. the initial enthusiasm has been considerably dampened by a Folk-theorem type result of Fudenberg and Maskin (1986). Their Folk theorem holds for all finitely and infinitely repeated games. It says that for any feasible, individually rational payoff vector there exists an ϵ-perturbation of the game under consideration such that this payoff vector can be sustained as the outcome of a sequential equilibrium of the perturbed game, if the players are patient enough and if there are enough repetitions. This theorem seems to devalue a whole literature. If any outcome can be explained by just picking the "appropriate" perturbation then the predictive power of a proposition based on such a perturbation is very limited indeed.[6)]

But what precisely is a "perturbation of the information structure of a game"? In Fudenberg and Maskin (1986) an ϵ-perturbation of the game means that with an arbitrarily small but positive probability (ϵ) some of the players are "crazy", i.e. they have a very different payoff function compared to the "sane" types. It has long been realized that the perturbation needed to sustain any particular payoff vector as a sequential equilibrium outcome is very specific, i.e. a very specific type of "crazyness" is needed. The question is whether this is the only sensible notion of what we mean by a perturbation of the information structure.

An alternative approach is to say that some perturbations are more plausible than others. One could argue that in a given economic situation there is some "natural" incomplete information. Although the players may know the game form and have a rough idea about each other's preferences they may not know precisely what the payoff functions of their opponents are. However, the possible types to

[6)] See Fudenberg (1990) for a discussion of the message of the Folk theorem which the other players attach positive probability may be close to each other. Thus, there is no crazy type with a very different payoff function but only a natural variety of different sane types. The meaning of "natural" cannot be defined a priori but depends on the economic problem under consideration.

This is the approach we follow in Chapter 2. There we consider a finitely repeated bargaining game in which a buyer and a seller have to trade one unit of a good or of a service in every period. While the valuation of the buyer is common knowledge (e.g. because she can buy the good for a given price somewhere else), the costs of the seller are assumed to be his private information. We allow for arbitrarily many different types (i.e. different possible costs) of the seller. In such a bargaining framework asymmetric information about the costs of the seller may be seen as a "natural" perturbation of the information structure of the game. Characterizing the set of all sequential equilibria satisfying a weak Markov property we can show a surprisingly strong result. In any such equilibrium the seller will never accept an offer lower than his highest possible costs in all but the last few periods of the game. The incomplete information ensures that he gets a substantially bigger share of the surplus than he would have got under complete information. This result holds almost independently of the discount factors of the two players. Even if the buyer is much more patient than the seller she will not try to price discriminate in all but perhaps the last (bounded number of) periods.

Our characterization argument given in Chapter 2 may be interesting for three reasons. First of all it demonstrates that part of the reasoning of Fudenberg and Levine (1989) can be extended to a two long-run player game. Second, the result is independent of the discount factors of the two players, so patience is not the crucial ingredient. Finally, our characterization follows a simple line of reasoning which provides a very intuitive understanding of the role of reputation. In fact we will show that it is not really "reputation" which matters along the equilibrium path, but that it may be more appropriate to call the effect “commitment through incomplete information”.

In Chapter 3 we take a more general approach to get out of the Folk theorem dilemma. If only “natural” incomplete information is considered, as in Chapter 2, the class of possible perturbations is very much restricted. In Chapter 3 we take the opposite direction and extend the class of possible perturbations. Following Fudenberg and Maskin (1986) suppose that with some probability each of the players is “crazy”. Yet, there may be many different kinds of crazyness in the world which cannot be excluded a priori. In Chapter 3 we allow for arbitrarily many and arbitrarily different possible types, including the types of crazyness considered by Fudenberg and Maskin (1986). This is in fact the approach Fudenberg and Levine (1989) have taken for the class of games in which a long-run player faces a sequence of short-run opponents. The question is whether their very powerful result can be generalized to the class of two long-run player games. In Chapter 3 we show that the answer is yes, if and only if the game is of “conflicting interests”. In a game of conflicting interests the strategy to which player one would most like to commit himself holds player two down to her minmax payoff. To give a few examples, this class of games includes the chain-store game with two long-run players, the war of attrition game, or a repeated sealed bid double auction. We can show that if there is a small prior probability that player one is a “commitment type” then he will get at least his “commitment payoff” in any Nash equilibrium of the repeated game if his discount factor approaches 1 (keeping player two’s discount factor fixed). This result is robust against further perturbations of the information structure, i.e. it holds independent of what other possible types are around with positive probability. This is in striking contrast to the message of the Folk-theorem of Fudenberg and Maskin (1986). Even if their “crazy” type exists with positive probability the “commitment” type will dominate the play. Thus we get a clearcut prediction of the equilibrium payoffs for all Nash equilibria of these games.

In both models of Chapter 2 and Chapter 3 commitment arises endougenously in equilibrium. An arbitrarily small but positive probability that one of the players is a "commitment type" can be used by this player to guarantee himself the payoff he would have got if he could have committed himself publicly always to play the strategy he prefers most.

1.3. The Incomplete Contracts Approach

While commitment may be achieved through a reputation effect in a repeated relationship, this approach cannot work if players move essentially only once, e.g. if they are engaged in jointly completing one big project which has no similar successors. Williamson (1975, 1985) and Klein, Crawford and Alchian (1978) have emphasized the importance of commitment in these situations to overcome the following problem. Suppose that in order to undertake a joint project some of the engaged parties have to make considerable "relationship specific" investments ex ante, i.e. investments the value of which is sharply reduced if they are not employed for this particular project. Once the investment costs are sunk these parties are (at least partially) locked in the relationship because the costs of the investment cannot be recovered by using it for a different purpose. If there are no safeguards against "opportunistic" behaviour these parties will be "held up" when it comes to the ex post bargaining on how to share the quasi-rents of the project.[7)] The division of the surplus, however, is not just a distributional matter but has an important impact on

[7)] Williamson (1975, 1985) introduced the term "opportunistic behaviour" which he defines as "self-interest seeking with guile". However, in common decision theoretic usage this is just rational (i.e. utility maximizing) behaviour. Similarly the "hold-up" problem just reflects the fact that the bargaining power of a player is sharply reduced after his investment costs are sunk since he cannot credibly threaten to opt out any more.

ex ante efficiency. If parties anticipate that their relationship specific investments will be exploited they will not invest efficiently. To overcome this problem it is necessary to commit ex ante how to share the quasi-rents emerging ex post, i.e. after investment costs have been sunk.

The classical way how to commit in this situation is to write a contract (enforceable by the courts) which specifies unambigiously how to divide the surplus in any possible contingency. If a complete contingent contract is feasible the commitment problem can be solved easily.

A complete contingent contract, however, is a fiction and can hardly be found in real economic situations. There are several reasons why most contracts necessarily have to be incomplete. First of all, parties cannot condition a contract on something which is not observable to all of them. For example, a wage contract cannot be made contingent on the effort a worker spends, if his effort level is only observed by himself. This problem is well known and has found considerable attention in the literature on moral hazard and adverse selection.[8)]

A more subtle problem is that although an event may be observable by all the contracting parties, it need not be "verifiable" to the courts. For example, all the parties engaged in a joint project may have observed that player A did not invest sufficiently in research and development for the project, but it may be impossible to prove this unambigiously to an outsider like the courts.

Finally it may be that the bargaining situation which arises ex post, after the parties have sunk their investments, cannot be ruled by a complete contingent contract ex ante because ex ante this situation is too complex. The optimal contract would have to be conditioned on the realization of the "state of the world" which materializes in course of the relationship. For example the precise characteristics of

[8)] For an excellent survey see Hart and Holmström (1987) the project may be unknown ex ante because they depend on the outcome of some research and development or on the ex post market conditions. Although players may be able to form a prior probability assessment over the possible states of the world and to decide rationally on their investment levels, it may be too complex or too costly to describe the possible contingencies unambigiously in an enforcable contract.[9)]

It is still an unsettled question whether in a situation with complete information the non-verifiability problem alone is sufficient to generate ex ante inefficient investments. If the investments are observable but not verifiable it may be possible to write a more sophisticated contract which does not condition on the investments or the possible states of the world directly but on verifiable messages the players contractually agree to exchange at some later point in time. To put it differently, players could commit in a contract to a mechanism (i.e. a game form) to be played in front of a court, where the mechanism is constructed such that the payoffs the players receive in equilibrium (depending on the realization of the state of the world) just give the right incentives to invest. The literature on mechanism design and implementation has shown that in "economic environments" almost everything can be implemented if the appropriate implementation concept is used, so first best investments can be induced.[10)] However, it is an open question which requirements the

9) The less formal literature, in particular Williamson (1985), claims that the ultimate reason for contract incompleteness is "bounded rationality". But if the rationality of the players is too bounded to write a complete contingent contract, why should they be able to rationally anticipate the returns of their investments in each state of the world? Without a comprehensive theory of boundedly rational behaviour (which does not exist up to now) there is no convincing answer to this question. This is why Hart (1990) insists that a theory of incomplete contracts should (and can) do without the ingredient of bounded rationality.

10) In an "economic environment" money can be used to give incentives. The use of money puts a restriction on the domain of the possible preferences of the players since it is assumed that everybody prefers more money to less. Without this restriction the implementation results are considerably weaker (see Moore and Repullo (1988) and Moore (1991)).

mechanism should satisfy (e.g. renegotiation proofness, robustness, etc., see Moore (1991)). This is why the literature on incomplete contracts frequently refers to non-verifiability and complexity to justify the assumption that ex ante no contingent contracts can be written at all, although it may be possible to contract on some variables ex post, after the state of the world has materialized.[11)]

However, it may be possible to write a contract ex ante on the "governance structure" (Williamson) of the project. A governance structure assigns control rights to the involved parties. The classical example is the assignment of ownership rights. Grossman and Hart (1986) define ownership as the residual right to control an asset in all contingencies which have not been dealt with in an explicit contract. The allocation of ownership rights affects the bargaining power of the players ex post and thus their expected share of the surplus to be devided. This idea can be used to explain the optimal allocation of ownership rights. Comparing the investment incentives of the players under different ownership structures it can be shown under which circumstances a vertically integrated firm will be more efficient than two separately owned companies. Thus, at least some commitment not to hold up each other after investment costs have been sunk can be achieved by assigning ownership rights appropriately.

In Chapter 4 we focus on a different commitment effect arising in a world of incomplete contracts. We argue that different allocations of ownership rights lead to different distributions of information. The distribution of information affects the ex post bargaining of the players in two ways. First of all, if a player has private information he will get an "information rent" in equilibrium, which increases his expected share of the surplus. Second, if there is asymmetric information, bargaining will no longer lead to an ex post efficient outcome. We demonstrate that both effects

[11)] See e.g. Grossman and Hart (1986), Hart and Moore (1990) or Bolton and Whinston (1990).

can be used to improve ex ante incentives. That is, through an appropriate choice of the allocation of ownership rights players can affect the information structure of the ex post bargaining game in order to improve ex ante investment incentives - which solves their commitment problem at least partially.

We develop this theoretical argument in a more applied context of economic policy. Consider the problem of a government which has to decide on whether to nationalize or to privatize a company. We argue that under privatization the government will have less access to inside information (e.g. about costs and profits) of the firm than under nationalization. The reason is that in the privatized firm the private owner has the residual rights of control, so he can always manipulate the information which is produced under his control to his advantage. If complete contracts are not feasible it is impossible to contract the right of direct access to this information to an outsider, so the private owner cannot sell it to the government.[12)]

Now suppose the government would like to commit ex ante to a subsidy scheme for the firm which gives some incentives for cost saving investments to the management. Under nationalization this scheme is not credible, because the government cannot commit not to expropriate the returns of the investment once the investment costs are sunk. This is Williamson's classical "hold up" problem. Under privatization however, the government is less informed about the costs of the firm. Therefore the optimal subsidy scheme under incomplete information gives an information rent to the private owner and distorts the optimal production level. We show how both effects can be used by the government to give better investment incentives to the management.

Thus, commitment has been achieved through the choice of a governance structure, i.e. through privatization. Note, however, that there is a trade off. Under privatization the management will invest more in cost reduction which increases ex

[12)] For a more detailed discussion of this point see Chapter 4.1.

ante efficiency, but - due to the asymmetric information - the production level will be distorted ex post, which is not the case under nationalization. This trade off may explain some of the costs and benefits of privatization.

Chapter 2:

Commitment through Incomplete Information in a Simple Repeated Bargaining Model

2.1. Introduction

Dynamic bargaining with one-sided asymmetric information about a single object is by now well understood. Suppose that a buyer who doesn't know the (opportunity) costs of a seller can make a take-it-or-leave-it offer for an indivisible durable good in every period. Her problem is that she cannot commit in the first period to a sequence of price offers and thus cannot prevent herself from raising the price after an offer has been rejected. Anticipating this the seller could just wait until he gets a more favourable offer. It has been shown that if the seller is very patient then the buyer loses almost any opportunity to price discriminate and offers a price close to the highest possible costs of the seller in the first period already.[1)]

Much less attention has been paid to the case of repeated bargaining where there is a perishable commodity or a service to be traded in every period. This situation differs substantially from the single trade bargaining problem in two respects. On the one hand the game doesn't end after a price offer has been accepted. Thus the seller might be reluctant to accept an offer if this reveals his costs to be low and gives him a smaller share of the surplus thereafter. This concern has been called "reputation effect" in other repeated games with asymmetric information. On the other hand, rejecting an offer is more costly than in the single trade bargaining case

[1)] See e.g. Sobel and Takahashi (1982), Fudenberg, Levine and Tirole (1985) or Gul, Sonnenschein and Wilson (1986). If the reservation value of the buyer strictly exceeds the highest possible costs of the seller, then there is a (generically) unique sequential equilibrium of the single trade bargaining game satisfying this property, which confirms the conjecture of Coase (1972).

because this period's trade is lost irretrievably. Suppose that the buyer is more patient than the seller, i.e. she gives less weight to losses today than her opponent. Then it may be worth her while to invest in "screening" the seller in the beginning of the game in order to get a higher expected share of the surplus thereafter.

In this chapter it is shown that the "reputation effect" strongly dominates the "screening effect". The seller will not accept any price offer lower than his highest possible costs before the very end of the game. This result holds almost independently of the discount factors. Even if the buyer (having a discount factor close to one) is much more patient than the seller (whose discount factor may be close to 0.5) she will not try to price discriminate in all but perhaps the last (bounded number of) periods. Thus, in the beginning of the game there is no transmission of information. This is in striking contrast to the equilibrium in the single trade bargaining game with repeated offers. There we have some price discrimination and transmission of information right from the beginning. Furthermore, the Coase conjecture holds only if the discount factor of the informed player (here the seller) is close to one, while discounting hardly matters in the repeated setting. Hence the bargaining position of the uninformed party is even worse under repeated bargaining.

This chapter is closely related to the literature on "reputation effects" started by Kreps and Wilson (1982a) and Milgrom and Roberts (1982). These authors have shown that a long-run player whose "type" (i.e. payoff function or strategy space) is private information can credibly threaten to behave as if he were another type, even if the prior probability of this type is very small.[2] However, there is an important methodological difference. In their papers and in most of the subsequent literature a sequential equilibrium is calculated explicitly while the method used in this chapter is to characterize equilibria. Without computing any equilibrium we can give a precise characterization of the equilibrium path and the equilibrium payoffs of all

[2] See the discussion of this literature in Chapter 1.2.

sequential equilibria satisfying a weak Markov property. The characterization approach has two main advantages. First, it allows for a more complex game structure with arbitrarily many different types and different discount factors for the seller and the buyer, while the calculation of an equilibrium is so complex that usually only two or three types can be considered. Second, our characterization follows a simple and intuitive line of reasoning which may provide a clearer understanding of the role of reputation.

Among the few other papers on equilibrium characterization in repeated games with incomplete information the one most closely related to ours is Fudenberg and Levine (1989). They allow for any perturbation of the information structure of the game which gives positive probability to a type playing a "commitment strategy" and they provide a lower bound for the equilibrium payoff of the informed player in any Nash equilibrium given that his discount factor is close to one. However, their result holds only if the uninformed player is completely myopic, or - equivalently - if the informed long-run player faces a sequence of uninformed short-run players, each of whom lives for one period only but knows the complete history of the game. We follow their approach in assuming that there exists a commitment type of the seller who accepts an offer if and only if it is greater or equal than the highest possible cost of the seller. All the other types of the seller differ only in their costs of production. Since this game has much more structure than the quite general class of games considered by Fudenberg and Levine (1989) we can extend the analysis to the two long-run player case. Furthermore, we can give a precise characterization of the equilibrium path, and our lower bound for the equilibrium payoffs is sharp for any discount factors bigger than 0.5 given that the horizon is long enough.

A repeated bargaining game similar to the one discussed here has been analyzed by Hart and Tirole (1988) who focus on the effects of long-term contracts and renegotiation. They consider the case of only two different types and common discount factors for both players. Both assumptions are crucial for their characterization argument to go through, and their proof follows a quite different line of reasoning.

The rest of this chapter is organized as follows. Section 2.2 describes the repeated bargaining game and introduces the notation. In Section 2.3 and 2.4 the main results are developed and discussed. Section 2.3 states some preliminary results which are used in Section 2.4 for the equilibrium characterization. The line of reasoning followed here gives a clear understanding of how "commitment through incomplete information" works. Section 2.5 deals with the existence problem. In Section 2.6 we conclude with some remarks on the transmission of information in equilibrium. It is shown that there is a close connection to the "ratchet effect" in games with a repeated principal-agent relationship. Then we analyze what is happening in the end of the game when the buyer starts to price discriminate. For the most simple case with only two different types of the seller we construct the (generically) unique equilibrium path. Finally, for the general case of arbitrarily many different types it is shown that information is not transmitted gradually (through successive skimming) as in a single trade bargaining game, but that equilibrium behaviour is extremely complex with both players randomizing in the last periods.

2.2. The Bargaining Game

A seller (S) can produce one unit of a good or of a service per time period with constant unit costs $c \leq 1$ which are his private information. The cost c may also be interpreted as his reservation value or his opportunity costs. In each period he wishes to sell the good to a buyer (B) with reservation price $b > 1$. Thus, trade is always efficient. The reservation price b is assumed to be common knowledge, e.g. because the buyer has the outside option to buy the good elsewhere for the fixed price b.

Before the first stage game starts the type of the seller is drawn by nature from the finite set $C = \{c^*, c^1, \cdots, c^n\}$, with $1 = c^* > c^1 > \cdots > c^n = 0$. It will be convenient to assume that all c^i are rational numbers. A cumulative distribution function $F(c)$ which is common knowledge represents the initial beliefs of the buyer about the type of the seller. $F(c)$ is the probability that his costs are less than or equal to c, and $\mu^i = F(c^i) - F(c^{i+1})$ denotes the prior probability that $c = c^i$, $i \in \{*, 1, \cdots, n\}$. Everything else, including the complete history of the game, is common knowledge.

The following stage game G takes place in every period: The buyer makes a "take-it-or-leave-it" offer $p_t \in P$, where the price space $P \subset \mathbb{R}$ is specified below in this section. If the seller accepts p_t (A) he produces the good with costs c and trade takes place. If he rejects (R) there is no production, no trade, and no possibility of further bargaining in period t. The stage game G repeated T times, $T < \infty$, is denoted by G^T. Time is counted backwards $(t = T, T-1, ..., 1)$ for convenience.

The stage game payoffs are $p_t - c^i$ for the seller with costs c^i $(i = 1, \cdots, n)$ and $b - p_t$ for the buyer if the seller accepts p_t and 0 for both of them if he rejects. The seller with type c^* is called the "commitment type" who has a slightly different payoff function. Following Fudenberg and Levine (1989) we assume that for him it is a dominant strategy in the repeated game to accept p_t if and only if $p_t \geq 1$. The idea is that there may be a small but positive probability that the seller is "committed" to accept price offers if and only if they are higher than a certain level which has been normalized to 1. We will show that the existence of such a type can be used by all the other types of sellers to credibly threaten to behave as if they were committed to this strategy as well in all but the last finite number of periods. The assumption of a "commitment type" is a little bit stronger than necessary. In the following analysis we only need that there is a type of the seller who follows this strategy in every equilibrium. In Section 2.6 we show for the special case with only two different types of sellers with the normal payoff functions that there is a generically unique sequential equilibrium in which the seller with cost of 1 follows this strategy. However, if there are more than two types, we cannot rule out the possibility that a seller with cost $c = 1$ rejects $p_t = 1$. This is why we assume that there may exist a commitment type with positive probability.[3)]

Let h_t be a specific history of the repeated game out of the set $H_t = \big(P \times \{A, R\}\big)^{T-t}$ of all possible histories up to but not including period t. A (behavioural) strategy of the buyer is a sequence of functions, $\sigma_t^B : H_t \to \Sigma^B$, each of which maps the set of all possible histories H_t into the set Σ^B of all probability density functions over the price space P. The seller with cost c conditions his plan of action in period t not only on h_t but also on the price offer p_t he has got in this period. Hence, a (behavioural) strategy of the seller is a sequence of functions $\sigma_t^S(c) : \big(H_t \times P\big) \to \Sigma^S$, where Σ^S is the set of all probability distributions over $\{A, R\}$. Finally, let $\mu_t^i(h_t)$, $i \in \{*, 1, \cdots, n\}$, denote the updated probability the buyer assigns to the event that $c = c^i$ given that she observed the history h_t. The reference to history will be omitted if there is no ambiguity.

All players are risk neutral. The seller (buyer) discounts his (her) payoff in future periods with the discount factor $\delta < 1$ ($\beta < 1$). The expected payoffs of the buyer and of the seller with cost c^i ($i = 1, \cdots, n$) from the beginning of period t for the rest of the game are given by

$$V_t^B \;=\; E_t \left\{\textstyle\sum_{j=0}^{t-1} \beta^j \cdot \pi_{t-j}(p_{t-j}) \cdot (b - p_{t-j})\right\}, \qquad (2.1)$$

3) An alternative specification is the following. Assume that the payoff function of type c^* in the stage game is given by

$$v_t(c^*, p_t) \;=\; \begin{cases} p_t - 1 - \theta \cdot \eta & \text{if } p_t \text{ is accepted} \\ 0 - (1 - \theta) \cdot \eta & \text{if } p_t \text{ is rejected} \end{cases}$$

where $\eta > 0$ may be arbitrarily small and $\theta = \begin{cases} 0 & \text{if } p_t \geq 1 \\ 1 & \text{if } p_t < 1 \end{cases}$. This payoff function is very close to the normal payoff function of a seller with cost of 1. It is easy to check that with this specification all our results still go through.

$$V_t^S(c^i) \;=\; E_t \;\left\{\sum_{j=0}^{t-1} \delta^j \cdot \pi_{t-j}(p_{t-j}) \cdot (p_{t-j} - c^i)|c^i\right\}, \tag{2.2}$$

where E_t is the expectation operator and $\pi_{t-j}(p_{t-j})$ denotes the probability that p_{t-j} is accepted in period $t-j$ given the history up to period $t-j$. To simplify notation the reference to history is omitted. The special case $\beta = 0$ is of particular interest. It is equivalent to the case of a sequence of buyers (B_t), each of whom knows the complete history of the game but lives for only one period and whose payoff function is

$$V_t^B \;=\; \pi_t(p_t) \cdot (b - p_t)\;. \tag{2.3}$$

With arbitrarily many different types of the seller and an arbitrary distribution function $F(c)$ it is extremely complex to calculate any sequential equilibrium. We compute an equilibrium explicitly only for the very special case with two possible types of the seller in Section 2.6. For the general case, however, we characterize rather than calculate the set of equilibria. Existence seems to be the main problem. In order to use a general existence theorem the buyer's strategy space, i.e. the price space P, has to be finite.[4)] However, the price grid has to be sufficiently fine compared to the horizon (T) of the game. Since we are interested in the limit behaviour of the set of equilibria if T goes to infinity, the parameters of the game have to be chosen in the following order. First, all parameters except T and P are fixed, then a large T is chosen and then a sufficiently fine price grid P (as compared to T) is selected.

For any given T we construct the price grid as follows: Fix a real number $\eta > 0$ which may be arbitrarily small. Recall that we assumed the possible costs of the seller to be rational numbers. Thus there exists an integer $\overline{r}$ and integers $k^*, k^1, \cdots k^n$ such that $c^i = \frac{k^i}{\overline{r}}$ and $\eta > \frac{1}{\overline{r}}$. Fix such an integer $\overline{r}$ and define $r = \overline{r} \cdot 2^T$. Finally

[4)] All our results (except existence) can be proved slightly more elegantly with a continuous price space. See Schmidt (1990a).

fix an integer $l > r \cdot b \cdot T$. Then the price grid in the game G^T is chosen to be

$$P(\eta, T) \;=\; \{p^0, p^1, \cdots p^l\}$$

where $p^j = \frac{j}{r}$, $j = 0, 1, \cdots, l$. Clearly, the buyer will never want to offer more than p^l, since $p^l > \frac{r \cdot b \cdot T}{r} = b \cdot T$, so the upper bound on prices is no restriction. If T increases the price grid becomes more and more close-meshed. Note that by construction the possible types of the seller are always on the grid.[5)]

The appropriate solution concept for this game with incomplete information is the notion of sequential equilibrium (SE) as developed by Kreps and Wilson (1982b). We provide a precise characterization of the equilibrium path and of the equilibrium payoffs of all sequential equilibria satisfying the following Weak Markov Property.

> **Definition:** A sequential equilibrium of G^T satisfies the "Weak Markov Property" (WMP) if the seller's equilibrium strategy σ_t^S in period t is contingent only on his type, the current price offer p_t, and the public beliefs μ_t in period t.

How severe is the restriction to Markov Perfection? In Section 2.5 we show that a sequential equilibrium satisfying WMP always exists. Thus, if along the equilibrium path the equilibrium strategies are unique then the Weak Markov Property holds along the equilibrium path in all sequential equilibria. Generic uniqueness can be shown for the simple case with only two different types of sellers. For this case it is possible to construct the unique equilibrium path and to see that the requirement of the Weak Markov Property is no restriction (see Section 2.6.2). Furthermore, in

5) This particular specification of the price grid will be convenient to work with in the proofs of the lemmata in the next section, but it is not necessary. We only have to require that the grid is "fine enough" for large T. For example, any grid $P' \supset P(\eta, T)$ would do the job as well. Nor is it necessary that the c^i are rational numbers lying on the grid, but it very much simplifies the exposition of the proofs.

the special case of a long-run seller facing a sequence of short-run buyers ($\beta = 0$) the restriction to equilibria satisfying the WMP is not necessary and all our results hold for all sequential equilibria. Thus the WMP has bite only if there are two long-run players with at least three different types of sellers. In this case, however, Markov Perfection is a sensible refinement restricting the set of equilibria, since it requires that the strategies of the players are as simple as is consistent with rationality. Furthermore it captures the idea that "bygones are bygones" more rigorously than does the notion of perfect equilibrium. That is, the outcome of a proper subform should not depend on the different paths by which this subform has been reached.[6] Finally, in this specific game, the Weak Markov Property gives a precise characterization of the equilibrium path and the equilibrium payoffs, as will be shown in the next sections.

2.3. Some Preliminary Results

The bargaining procedure is modelled in such a way that in each period all the bargaining power is on the side of the uninformed buyer. If she knew the cost of the seller she could extract all the surplus of trade by offering $p_t = c$ which would always be accepted in equilibrium. However, the seller's cost is private information and he may use this advantage to get some share of the surplus as an information rent. The following Lemmata 2.1 to 2.3 give a first characterization of the buyer's and the seller's behaviour in equilibrium. Lemma 2.1 provides an upper bound for the prices offered in equilibrium and a lower bound for the equilibrium payoff of the buyer.

In the following it will be convenient to identify the commitment type of the seller with $c^* = 1$. In Lemmata 2.1 and 2.2 it doesn't matter whether type c^* is

[6] For a more detailed discussion of these points see Maskin and Tirole (1989).

a commitment type or whether he has a normal payoff function with cost $c = 1$. Recall from the definition of the price grid $P(\eta, T)$ that for any arbitrarily small η the integer r has been chosen such that $\eta > \frac{2^T}{r}$.

Lemma 2.1: Consider any history up to but not including period t of G^T, and let c_t^m denote the maximum of all types to whom the buyer assigns positive probability after this history. If c^* has positive probability, then $c_t^m = 1$. On the equilibrium path of every SE satisfying the WMP all types $c \leq c_t^m$ will accept any price offer $p_t \geq c_t^m + \frac{2^{t-1}}{r}$ and the buyer will offer $p_t \leq c_t^m + \frac{2^{t-1}}{r}$. The expected equilbrium payoff of the buyer from period t onwards is bounded below by

$$\underline{V}_t^B(c_t^m) \;=\; \sum_{j=0}^{t-1} \beta^j \cdot \left(b - c_t^m - \frac{2^j}{r} \right) \;>\; \sum_{j=0}^{t-1} \beta^j \cdot (b - c_t^m) - \eta \; . \tag{2.4}$$

<u>Proof:</u> The proof is by induction. In the last period ($t = 1$) the buyer believes that $c \leq c_1^m$ with probability 1. If $p_1 > c_1^m$ is offered every $c \leq c_1^m$ strictly prefers to accept p_1. Thus, $p_1 \leq c_1^m + \frac{1}{r}$ will be offered in equilibrium. The buyer can guarantee herself a payoff of at least $\underline{V}_1^B(c_1^m) = b - c_1^m - \frac{1}{r} > b - c_1^m - \eta$ by just offering $p_1 = c_1^m + \frac{1}{r}$.

Suppose the lemma is true for period t. In period $t+1$ the upper bound of the buyer's beliefs about c is c_{t+1}^m. Suppose $p_{t+1} \geq c_{t+1}^m + \frac{2^t}{r}$ and there are some $c \leq c_{t+1}^m$ who reject p_{t+1} with positive probability. Take the maximum of these c and call it $\overline{c}$. If $\overline{c}$ rejects he gets nothing in period $t + 1$ and he has revealed that his costs are at most $\overline{c}$. Hence $c_{t-j}^m \leq \overline{c}$ ($j = 0, \cdots, t-1$), because the updating of the beliefs has to be consistent in the equilibrium which is supposed to be played from this subform onwards. However by the induction hypothesis the seller won't be offered more than $c_{t-j}^m + \frac{2^{t-j-1}}{r}$ in the future. Thus, the overall payoff of type $\overline{c}$ from rejecting p_{t+1} is at most

$$\delta \cdot \sum_{j=0}^{t-1} \delta^j \cdot \frac{2^{t-j-1}}{r} \;<\; \sum_{j=0}^{t-1} \frac{2^{t-j-1}}{r} \;=\; \frac{2^t - 1}{r} \;. \qquad (2.5)$$

If he accepts p_{t+1} his payoff is at least $p_{t+1} - \bar{c} \geq \frac{2^t}{r}$, so $\bar{c}$ would have done better accepting. Therefore such a $\bar{c}$ does not exist and every $c \leq c^m_{t+1}$ will accept $p_{t+1} \geq c^m_{t+1} + \frac{2^t}{r}$.

Suppose the buyer offers $p_{t+1} > c^m_{t+1} + \frac{2^t}{r}$ in equilibrium. Since any $p_{t+1} > c^m_{t+1} + \frac{2^t}{r}$ will be accepted by all $c \leq c^m_{t+1}$ any such p_{t+1} will yield the same posterior beliefs about c in period t and thus, by the Weak Markov Property, the same equilibrium payoff for the buyer from period t onwards. Therefore $p_{t+1} > c^m_{t+1} + \frac{2^t}{r}$ cannot be an equilibrium price offer, and the buyer will offer $p_{t+1} \leq c^m_{t+1} + \frac{2^t}{r}$. Suppose the buyer offers $p_j = c^m_{t+1} + \frac{2^{j-1}}{r}$ in period j, $j = t+1, \cdots, 1$. By the induction hypothesis all these price offers will be accepted. Thus the buyer can guarantee herself at least

$$\begin{aligned} V^B_{t+1}(c^m_{t+1}) \;&=\; \sum_{j=0}^{t} \beta^j \cdot (b - c^m_{t+1} - \frac{2^j}{r}) \;=\; \sum_{j=0}^{t} \beta^j \cdot (b - c^m_{t+1}) \;-\; \sum_{j=0}^{t} \beta^j \cdot \frac{2^j}{r} \\ &>\; \sum_{j=0}^{t} \beta^j \cdot (b - c^m_{t+1}) \;-\; \frac{2^{t+1}}{r} \;\geq\; \sum_{j=0}^{t} \beta^j \cdot (b - c^m_{t+1}) \;-\; \eta \;. \end{aligned} \qquad (2.6)$$

Q.E.D.

Note that Lemma 2.1 rests on a backward induction argument which can't be used if there is an infinite horizon. In fact, it is easy to construct an equilibrium of the infinitely repeated game where Lemma 2.1 does not hold. Note further that the Weak Markov Property has only been used in the induction step to show that the buyer will never offer $p_{t+1} > c^m_{t+1} + \frac{2^t}{r}$. It is easy to see that in the case of a long-run seller facing a sequence of short-run buyers ($\beta = 0$) this follows trivially from the fact that the buyer in period $t+1$ is only interested in her payoff of the current period. Given that any $p_{t+1} \geq c^m_{t+1} + \frac{2^t}{r}$ will be accepted with probability 1, a short-run buyer will never offer more than $c^m_{t+1} + \frac{2^t}{r}$. Therefore we don't need the WMP in this case. However, in the more general case of two long-run players

($\beta > 0$) the buyer might offer $p_{t+1} > c^m_{t+1} + \frac{2^t}{r}$. Although this loses money in period $t+1$ he might be compensated in the future if this price offer leads to a continuation equilibrium which gives the buyer a higher payoff than he would have got otherwise.[7] To deal with this problem the WMP is needed. All the following lemmata and propositions use Lemma 2.1. Thus, if not stated otherwise, from now on we mean by "equilibrium" a sequential equilibrium satisfying the WMP.

Lemma 2.2 says that it never pays for the seller to accept an offer significantly lower than his costs.

Lemma 2.2: If p_t is offered in period t, every seller with type $c \geq p_t + \frac{2^{t-1}}{r}$ strictly prefers to reject p_t.

Proof: Suppose there are some types $c \geq p_t + \frac{2^{t-1}}{r}$ who accept p_t with positive probability. Take the maximum of these types and denote it by $\bar{c}$. If the seller with costs $\bar{c}$ accepts p_t his payoff in period t is $p_t - \bar{c} \leq -\frac{2^{t-1}}{r}$ and he has revealed that his costs are $c \leq \bar{c}$. By Lemma 2.1 he won't be offered more than $p_j = \bar{c} + \frac{2^{j-1}}{r}$ ($j = t-1, \cdots, 1$) in the future. So the most he can gain in the future is

$$\sum_{j=1}^{t-1} \delta^j \cdot \frac{2^{j-1}}{r} < \sum_{j=1}^{t-1} \frac{2^{j-1}}{r} < \frac{2^{t-1}}{r} . \tag{2.7}$$

Therefore he can't be compensated for his loss and strictly prefers to reject p_t.

Q.E.D.

The special payoff function of the commitment type is exploited for the first time in the following Lemma 2.3, which states that $p_t \geq 1$ will always be accepted in equilibrium.

[7] I am grateful to Drew Fudenberg and an anonymous referee who made me aware of this problem.

Lemma 2.3: If $p_t \geq 1$ is offered it will be accepted with probability 1 in any equilibrium. Furthermore, if $c_t^m = 1$ the equilibrium payoff of the buyer in any subform is bounded below by $\underline{V}^B = \sum_{j=0}^{t-1} \beta^j \cdot (b-1)$.

Proof: The commitment type c^* accepts $p_t \geq 1$ by assumption. Suppose there is a c^i, $i \in \{1, \cdots, n\}$, who rejects $p_t \geq 1$ in equilibrium with positive probability. Denote the maximal such type by $\overline{c}$. His payoff in period t is 0 and he has revealed that his costs are at most $\overline{c}$. From Lemma 2.1 we know that he won't be offered more than $\overline{c} + \frac{2^{j-1}}{r}$ in periods $j = t-1, \cdots, 1$. So his overall payoff is bounded above by $\frac{2^{t-1}}{r} < \eta$. However, by accepting he could have got $1 - \overline{c} \geq 1 - c^1 > \eta$, a contradiction. It follows immediately that the buyer can guarantee herself at least $\underline{V}^B = \sum_{j=0}^{t-1} \beta^j \cdot (b-1)$ by just offering $p_t = 1$ in every period. *Q.E.D.*

Lemma 2.3 implies that if the commitment type of the seller has positive probability, then in any equilibrium there exists with positive probability a history in which p_t is accepted if and only if $p_t \geq 1$.

Lemma 2.4 focuses on such a history and deals with the following question which is fundamental for the characterization results of the next section: Suppose the buyer offers $p_t < 1$ in equilibrium. Is there a lower bound $\epsilon > 0$ independent of t, such that the probability of acceptance of p_t, denoted by $\pi_t(p_t)$, must be at least ϵ? Such a lower bound clearly exists in the special case of a sequence of short-run buyers, each of whom lives for one period only ($\beta = 0$). By Lemma 2.3 a lower bound for the equilibrium payoff of a short-run buyer is given by $\underline{V}^B = b - 1$. Thus, if she offers $p_t < 1$ in equilibrium it must be true that $\pi_t(p_t) \cdot (b - p_t) \geq b - 1$, or

$$\pi_t(p_t) \geq \frac{b-1}{b-p_t} \geq \frac{b-1}{b} > 0. \tag{2.8}$$

However, in the general case of a long-run buyer ($\beta > 0$) this need no longer be true. A long-run buyer is concerned about both her present and her future payoff, so she may trade off losses today against gains tomorrow. In particular, it might pay for her to invest in gathering information in the beginning of the game, i.e. to "test" the type of the seller with price offers lower than 1 even if the probability of acceptance is very small. Lemma 2.4 says that although for any $p_t < 1$ the probability of acceptance may be arbitrarily small it cannot be arbitrarily small for arbitrarily many price offers.

Lemma 2.4: Assume $\mu^* > 0$. Fix an integer M,

$$M \;\geq\; N \;=\; \frac{ln(1-\beta) + ln(b-1) - ln\, b}{ln\, \beta} \;>\; 0, \tag{2.9}$$

and define a positive number ϵ,

$$\epsilon \;=\; \frac{(1-\beta)\cdot(b-1)}{b} - \beta^M \;>\; 0\ . \tag{2.10}$$

Consider a history h_t of any equilibrium in which all price offers p_j ($j = T, \cdots, t$) have been accepted if and only if $p_j \geq 1$. If there have been M price offers $p_j < 1$ along such a history, at least one of them must have had a probability of acceptance of at least ϵ.

The proof is lengthy and relegated to the appendix but the underlying intuition can be outlined briefly. By Lemma 2.3 we know that $\underline{V}_t^B = \sum_{j=0}^{t-1} \beta^j \cdot (b-1)$ is a lower bound for the equilibrium payoff of the buyer. Thus, offering $p_t < 1$ must yield an expected payoff of at least $\underline{V}_t^B$. On the other hand $\overline{V}_t^B = \sum_{j=0}^{t-1} \beta^j \cdot b$ clearly is an upper bound for her equilibrium payoff after any history h_t because the seller will guarantee himself at least 0. Now suppose the buyer "invests" in screening the different types of sellers by making M price offers smaller than 1 although she believes that each of them will be rejected with a probability higher than $1 - \epsilon$. The possible return of such an investment is bounded above by $\overline{V}_t^B$. However, if the buyer discounts her future payoffs, the return from an investment may not be delayed too far to the future. But ϵ and M have been constructed so that when the buyer makes M price offers smaller than 1, each of them having a probability of acceptance of less than ϵ, then her highest possible expected payoff is smaller than $\underline{V}_t^B$, a contradiction.

2.4. Equilibrium Characterization

The following proposition states that if all offers $p_t < 1$ have been rejected then the buyer will offer prices lower than 1 at most K times in equilibrium, where K is a finite number depending on β, b, and μ^* but independent of T. Thereafter she will offer $p_t = 1$ until the game ends. The intuition behind this result is simple and gives a clear understanding of how the building of reputation works: If the buyer repeatedly offers prices smaller than 1 she ultimately expects that some of these prices will be accepted with positive probability (this is the role of Lemma 2.4). However, only a seller with $c < 1$ may accept $p_t < 1$. Hence, if $p_t < 1$ is rejected, the updated probability that $c = c^*$ increases. If the buyer has expected p_t to be accepted with a probability higher than $\epsilon > 0$ then the updated probability that $c = c^*$ after p_t has been rejected has to increase by an amount bounded away from 0. But the updated probability that $c = c^*$ is bounded above by 1. Therefore this can happen only in a bounded number of periods. In other words: If the seller has rejected price offers lower than 1 often enough then the updated probability that he is the commitment type becomes so big that the buyer will no longer try to price discriminate. This argument is made precise in the proof of the following proposition.[8)]

[8)] The part on Bayesian updating in the proof of Proposition 2.1 is similar to the proof of Lemma 1 in Fudenberg and Levine (1989). See Chapter 3.2, Lemma 3.1. However, in the setting of our model the argument is simpler and has been developed independently.

Proposition 2.1: Let $0 \leq \beta < 1$, $0 \leq \delta \leq 1$, $b > 1$, $\mu^* > 0$, and let $[x]$ denote the integer part of x. Then along every equilibrium path of G^T the following property holds: For every history in which an offer p was accepted if and only if $p \geq 1$ there could have been at most K offers $p < 1$, where

$$K = K(\beta, b, \mu^*) = M \cdot \left(\left[\frac{ln\, \mu^*}{ln \left(\frac{\beta b - \beta + 1}{b} + \beta^M \right)} \right] + 1 \right) \ , \tag{2.11}$$

with

$$M \ = \ [N] + 1 \ = \ \left[\frac{ln(1-\beta) + ln(b-1) - ln\, b}{ln\, \beta} \right] + 1 \ \geq \ 1 \ . \tag{2.12}$$

<u>Proof:</u> Assume there is an equilibrium of G^T for which the property does not hold, i.e. there is a history h^* along the equilibrium path in which the seller has accepted p_t iff $p_t \geq 1$ $(t = T, \cdots, 1)$ and in which the buyer has offered $p_t < 1$ more often than K times. We will show that this leads to a contradiction. Recall that $\mu_t^* = \mu_t^*(h_t)$ denotes the updated probability that $c = c^*$ given history h_t. By Lemma 2.3 we know that prices bigger or equal than 1 will be accepted by all types, so the probability assessment remains unchanged if $p_t \geq 1$ has been accepted. Prices lower than 1 will be rejected for sure by the seller with type c^*. Therefore along the history h_t^* the updated probability $\mu_t^*(h_t^*)$ cannot decrease, where h_t^* is the history h^* truncated at period t.

Devide the K (or more) price offers lower than 1 to successive groups of M, $M = [N] + 1 > N = \frac{ln(1-\beta)+ln(b-1)-ln\, b}{ln\, \beta} \geq 0$ and consider the first group of M of them. By Lemma 2.4 we know that at least one of them (call it p_{τ_1}) must have a probability of acceptance of at least $\epsilon = \frac{(1-\beta)(b-1)}{b} - \beta^M > 0$. Therefore the updated probability that $c = c^*$ after p_{τ_1} has been rejected is:

$$\begin{aligned}
\mu^*_{\tau_1 - 1} &= \text{Prob}(c = c^* \mid p_{\tau_1} \text{ was rejected in } \tau_1) \\
&= \frac{\text{Prob }(c = c^* \text{ and } p_{\tau_1} \text{ was rejected in } \tau_1)}{\text{Prob }(p_{\tau_1} \text{ was rejected in } \tau_1)} \\
&= \frac{\mu^*_{\tau_1}}{1 - \pi_{\tau_1}(p_{\tau_1})} \geq \frac{\mu^*_{\tau_1}}{1-\epsilon} \geq \frac{\mu^*}{1-\epsilon}.
\end{aligned} \tag{2.13}$$

Now take the second group of M price offers lower than 1. Again at least one of them (p_{τ_2}) has a probability of acceptance higher than ϵ. This gives

$$\mu^*_{\tau_2 - 1} \geq \frac{\mu^*_{\tau_2}}{1-\epsilon} \geq \frac{\mu^*_{\tau_1 - 1}}{1-\epsilon} \geq \frac{\mu^*}{(1-\epsilon)^2}. \tag{2.14}$$

Consider the nth group of M price offers lower than 1 and denote by p_{τ_n} one of the offers with a probability of acceptance higher than ϵ. By induction the updated probability ($\mu^*_{\tau_n - 1}$) that $c = c^*$ after p_{τ_n} has been rejected is bounded below:

$$\mu^*_{\tau_n - 1} \geq \frac{\mu^*}{(1-\epsilon)^n}. \tag{2.15}$$

However, $\mu^*_{\tau_n - 1} \leq 1$. Therefore there is an upper bound for n, namely

$$n \leq \frac{ln\, \mu^*}{ln(1-\epsilon)}. \tag{2.16}$$

Substituting for ϵ gives

$$n \leq \frac{ln\, \mu^*}{ln\left(\frac{\beta b - \beta + 1}{b} + \beta^M\right)}. \tag{2.17}$$

Thus, an upper bound for the number of periods in which price offers smaller than 1 are offered and rejected on the equilibrium path is given by

$$K(\beta, b, \mu^*) = M \cdot \left[\frac{ln\, \mu^*}{ln\left(\frac{\beta b - \beta + 1}{b} + \beta^M\right)} + 1\right]. \tag{2.18}$$

Note that K is increasing in M. Thus, choosing $M = [N] + 1$ minimizes K. Thus we have a contradiction to the assumption that there are more than K such price offers. *Q.E.D.*

It is evident from the proof how K depends on μ^*, b, and β. The bigger μ^* the less updating is needed to hit the point where $\mu^*_{\tau_n - 1}$ would become higher than 1, and the smaller is K. Furthermore K is a decreasing function of b (note that the numerator and the denominator are both negative). The reason is that the lower bound for the equilibrium payoff of the buyer is increasing with b. Thus, if she risks price offers smaller than 1 the possible loss is bigger and therefore the probability of acceptance must be higher. This gives a bigger ϵ and a smaller K. Finally K is an increasing function of β. If the buyer is more patient, then future gains become more important and she may offer prices lower than 1 with a smaller probability of acceptance in order to invest in gathering information. This effect increases the number of periods in which prices lower than 1 can be offered and rejected. On the other hand, if $\beta \to 0$ it is easy to see that $N \to 0$ and

$$K(0, b, \mu^*) \;=\; \left[-\,\frac{\ln \mu^*}{\ln b} \right]. \tag{2.19}$$

This is the same upper bound as the one derived by Fudenberg and Levine (1989) for the special case of an informed long-run player facing a sequence of uninformed short-run players.[9)]

Proposition 2.1 has an immediate implication for the equilibrium payoff of the seller. Suppose a seller with type $c < 1$ decides to build up a "highest cost" reputation and mimics the commitment type. The worst that can happen to him is that K price offers lower than 1 are offered and have to be rejected and that this

9) Fudenberg and Levine (1989) argued that their results do not apply to the two long-run player case because an uninformed long-run player may take an action in equilibrium which is not a short-run best response to the "commitment strategy" of the informed player, even if the probability that the "commitment strategy" will be played is arbitrarily close to 1. This is certainly true. However, our argument here is that this may not happen arbitrarily often if the uninformed player discounts her future payoffs and if her future payoffs are bounded above (see Lemma 2.4), which gives the positive lower bound for the probability of acceptance in at least some periods. This argument is not restricted to the special repeated bargaining game considered here, but holds in all two long-run player games with "conflicting interests". This class of games is discussed extensively in Chapter 3.

happens in the first K periods of the game. In this case he would get 0 in the first K periods and $1 - c$ thereafter.

Corollary 2.1: In the game G^T a lower bound for the equilibrium payoff of the seller with type c is given by

$$\underline{V}^S(c) \;=\; \sum_{j=0}^{K-1} \delta^j \cdot 0 + \sum_{j=K}^{T-1} \delta^j \cdot (1-c) \;=\; (1-c) \cdot \delta^K \cdot \frac{1-\delta^{T-K-1}}{1-\delta}\,. \tag{2.20}$$

However, this lower bound need not be very informative. Depending on β, b, and μ^* it may take quite a long time until the buyer starts to offer $p = 1$. Therefore, this lower bound gives a precise characterization of equilibrium payoffs only if the seller is very patient (δ is close to 1) and if the time horizon is sufficiently long.

Consider again the interpretation of Proposition 2.1. Suppose a seller with $c < 1$ mimics the commitment type. It follows from Proposition 2.1 that there will be at most $K(\beta, b, \mu^*)$ price offers lower than 1. If the seller rejects $p_t < 1$ often enough the probability that he is of type c^* increases until it becomes so big that the buyer will no longer try to price discriminate. In this sense the seller may invest repeatedly in the reputation of being the commitment type (i.e. in the updated probability that he is of type c^*). The costs of this investment are the missed opportunities of trade. The returns are the price offers of 1, which he will get for sure after K such rejections. Thus, Proposition 2.1 tells us that in principle every type of seller can build up this reputation. The question arises whether there are equilibria of the game with some types of sellers investing into this reputation in the beginning of the game in order to get better price offers thereafter. The answer is no, for the paradoxical reason that the incentives to do so are too big. Every type of seller would want to build up this reputation as long as the end of the game is not too close. Hence, every type would reject price offers lower than 1, but anticipating this the buyer will offer $p_t = 1$ in all periods before the very end of the game. Therefore the type of the seller will not be tested in the beginning of the game, the probability that he is of type c^* will remain at μ^*, and there is no reputation building in all but perhaps some of the last finite number of periods.[10)]

Proposition 2.2: Let β, δ, b, and μ^* satisfy: $0 \leq \beta < 1$, $0.5 < \delta \leq 1$, $b > 1$, and $\mu^* > 0$. Define $L = L(\delta)$ as the smallest integer such that $\sum_{j=1}^{L} \delta^j > 1$ and choose η such that

$$\sum_{j=1}^{L} \delta^j \;\geq\; 1 + \frac{\eta}{1 - c^1} \;. \tag{2.21}$$

If $T > K(\beta, b, \mu^*) \cdot L(\delta)$ then along the equilibrium path of every equilibrium of G^T a price of 1 is offered in at least the $T - K \cdot L$ first periods.

<u>Proof:</u> Take any equilibrium and fix a history h^* in which the seller has accepted p iff $p \geq 1$. From Proposition 2.1 it is known that after at most K rejections of price offers lower than 1 the buyer will offer a price of 1 from then on. Suppose there have been $K - 1$ such rejections up to period τ_1 and that there are more than L periods left ($\tau_1 > L$). If $p_{\tau_1} < 1$ is offered and rejected the payoff of the seller from period τ_1 onwards is

$$V_{\tau_1}(c \mid p_{\tau_1} R) = 0 + \sum_{j=1}^{\tau_1 - 1} \delta^j \cdot (1 - c). \tag{2.22}$$

Using Lemma 2.1 the highest payoff he can expect if he accepts p_{τ_1} is given by

$$\overline{V}_{\tau_1}(c \mid p_{\tau_1} A) = p_{\tau_1} - c + \sum_{j=1}^{\tau_1 - 1} \delta^j \cdot (\bar{c} + \frac{2^{\tau_1 - j - 1}}{r} - c) \;, \tag{2.23}$$

where $\bar{c}$ is the maximum of all types who would accept p_{τ_1} with positive probability. For type $\bar{c}$ it must be true that

$$p_{\tau_1} - \bar{c} + \sum_{j=1}^{\tau_1 - 1} \delta^j \cdot \frac{2^{\tau_1 - j - 1}}{r} \;\geq\; (1 - \bar{c}) \cdot \sum_{j=1}^{\tau_1 - 1} \delta^j \;. \tag{2.24}$$

10) Hart and Tirole (1988, Proposition 3) provide a similar but more restricted result.

But this is a contradiction because

$$
\begin{aligned}
p_{\tau_1} - \overline{c} + \sum_{j=1}^{\tau_1-1} \delta^j \cdot \frac{2^{\tau_1-j-1}}{r} \ &< \ 1 - \overline{c} + \frac{2^{\tau_1-1}}{r} \ < \ 1 - \overline{c} + \eta \\
&= (1-\overline{c}) \cdot \left(1 + \frac{\eta}{1-\overline{c}}\right) \leq (1-\overline{c}) \cdot \left(1 + \frac{\eta}{1-c^1}\right) \leq (1-\overline{c}) \cdot \sum_{j=1}^{\tau_1-1} \delta^j \ .
\end{aligned}
\tag{2.25}
$$

Therefore there is no such $\overline{c}$ and all types will reject any $p_{\tau_1} < 1$.

Suppose the buyer would offer $p_{\tau_1} < 1$ in equilibrium nevertheless. Then his payoff from τ_1 onwards is given by

$$V_{\tau_1}^B(p_{\tau_1}) = 0 + \beta \cdot V_{\tau_1-1}^B(p_{\tau_1}R). \tag{2.26}$$

By Lemma 2.3 a price offer $\tilde{p}_{\tau_1}$ with $1 \leq \tilde{p}_{\tau_1} < b$ would have been accepted with probability 1 yielding a payoff

$$V_{\tau_1}^B(\tilde{p}_{\tau_1}) = b - \tilde{p}_{\tau_1} + \beta \cdot V_{\tau_1-1}^B(\tilde{p}_{\tau_1}A). \tag{2.27}$$

However, nothing is learnt about the type of the seller neither after p_{τ_1} has been rejected nor after $\tilde{p}_{\tau_1}$ has been accepted and the probability assessment of the buyer is the same after both price offers. Thus, by the WMP, her equilibrium payoff in both subforms has to be the same and $V_{\tau_1}^B(\tilde{p}_{\tau_1}) > V_{\tau_1}^B(p_{\tau_1})$, a contradiction. Therefore, the buyer will not offer $p_{\tau_1} < 1$ in equilibrium.

We can go on by backward induction: Suppose there have been $K-2$ rejections of price offers lower than 1 up to a period τ_2. If $\tau_2 > 2 \cdot L$, then every seller will get price offers of 1 at least in the next L periods if he rejects, and because of the above argument every type will do so. But then the buyer will offer a price of 1 in this period already. The proof goes on by induction until there have been $K - K = 0$ rejections of price offers lower than 1. *Q.E.D.*

Note that the WMP is necessary (as in Lemma 2.1) to deal with the problem of multiple equilibria giving different continuation payoffs to the buyer in identical subforms. However, if the buyer is only interested in her payoff of the current period ($\beta = 0$) this restriction is not necessary and Proposition 2.2 holds for all sequential equilibria.

Proposition 2.2 is rather insensitive to variations of δ. If $\delta \in [0.62, 1)$ then $L(\delta) = 2$, i.e. independent of the patience of the seller a price of 1 will be offered in the first $T - 2 \cdot K$ periods of the game. For large T this gives a sharp lower bound for the equilibrium payoff of the seller independent of δ. This is quite surprising given the earlier lower bound $\underline{V}^S(c)$ of Corollary 2.1 which is sharp only if δ is very close to 1.

Suppose that before the beginning of period T *every* type of the seller could publicly commit never to accept an offer smaller than 1. In this "commitment version" of G^T the unique equilibrium, which will be called the "commitment equilibrium", prescribes that the buyer always offers $p_t = 1$ which the seller will always accept. Given this definition we can state our main result, which follows directly from Propositions 2.1 and 2.2:

Theorem 2.1: Let β, δ, b and μ^* satisfy: $0 \leq \beta < 1$, $0.5 \leq \delta \leq 1$, $b > 1$, and $\mu^* > 0$. Consider a sequence of games G^T. If T goes to infinity the equilibrium paths of all equilibria of G^T converge to the commitment equilibrium path. That is, there are finite numbers $K = K(\beta, b, \mu^*)$ and $L = L(\delta)$ independent of T such that for all T and $t \geq T - K \cdot L$ the buyer offers $p_t = 1$. If T goes to infinity the equilibrium payoffs of the buyer and the seller converge to

$$V_\infty^B = \frac{b-1}{1-\beta} \quad \text{and} \quad V_\infty^S = \frac{1-c}{1-\delta}. \tag{2.28}$$

Theorem 2.1 gives a precise characterization of the equilibrium path and the equilibrium payoffs for large T. Perhaps surprisingly, this result holds even if the buyer is much more patient than the seller, as long as $\beta < 1$ and $\delta > 0.5$. Intuitively one might have expected that a patient uninformed buyer who deals with a relatively impatient informed seller will try to screen the different types sellers with price offers lower than 1 in early periods in order to get more of the surplus of trade thereafter. Theorem 2.1 states that this is not the case. The reason is that as long as $\delta > 0.5$ every type of seller has a strong incentive to build up the reputation of being the commitment type. Therefore everybody will reject $p_t < 1$ in all but the last finite number of periods. But then there is no way to screen the different types and the buyer does better by offering 1, even if she is very patient. Note however, that there is no actual "reputation building" on the equilibrium path, i.e. the updated probability that $c = c^*$ remains unchanged until the very end of the game. It is the mere possibility to build up a reputation which prevents the buyer from offering prices lower than 1. Thus it is perhaps more accurate not to talk about "reputation building" but to say that the seller can use the incomplete information about his type to credibly threaten to behave as if he were the commitment type. This is why we call this effect "commitment through incomplete information".

2.5. Existence

Up to now all sequential equilibria satisfying the WMP have been characterized without showing that there exists at least one such equilibrium. For very simple cases (e.g. if there are only two different types of the seller and if there is no discounting, see Section 2.6.2) it is possible to show existence (and generic uniqueness) by constructing the equilibrium path. But for arbitrarily many different types and different discount factors this task is hopelessly complicated and a general existence theorem has to be used. By Theorem 6 of Selten (1975) and Proposition 1 of Kreps and Wilson (1982b) there exists a sequential equilibrium in G^T. However, if $\beta > 0$

we have to show that there is a sequential equilibrium of G^T satisfying the Weak Markov Property. Recall that the WMP restricts only the equilibrium strategy of the informed party (the seller) but not of the uninformed player. The WMP is clearly satisfied in a "strong Markov equilibrium", i.e. in a sequential equilibrium in which the (behavioural) strategies of both players depend only on the current beliefs and on the current price offer p_t, but it is well known that strong Markov equilibria do not in general exist.[11] However, in a recent paper Maskin and Tirole (1989) have shown existence for a slightly weaker equilibrium concept which they call "Markov Perfect Bayesian Equilibrium" (MPBE). We can state the following corollary to their Proposition 9.

Corollary 2.2: There exists a Markov Perfect Bayesian Equilibrium in G^T. Every MPBE is a sequential equilibrium.

Although the definition of a MPBE is quite complex and not provided here it is easy to show that in G^T every MPBE satisfies the Weak Markov Property. Thus there always exists at least one equilibrium which can be characterized as in Section 2.4.

2.6. Transmission of Information in Equilibrium

The equilibrium characterization of Section 2.4 has shown that the seller cannot be induced to transmit any information about his costs to the buyer in the beginning of the repeated relationship. This confirms the so called "ratchet effect", which has been formalized by Freixas, Guesnerie and Tirole (1985) and Laffont and Tirole (1988) in a principal-agent stage game which is played in two successive periods. The literature on the ratchet effect offers a slightly different interpretation of our

[11] See e.g. Kreps and Wilson (1982a) or Fudenberg and Tirole (1983). In their non-existence examples it is always the uninformed party who has to condition her strategy on past actions in equilibrium.

results which focuses on the information rents of the informed party and which is helpful to understand the necessary ingredients of our model. Thereafter we show how information is transmitted in the end of the game when the buyer starts to price discriminate. We conclude with some remarks on a possible extension of the model.

2.6.1. The Ratchet Effect Revisited

Laffont and Tirole (1988) show that an agent with (privately known) high productivity has a strong incentive to pool with the low productivity type in order to hide his ability in the first period. The reason is that the principal cannot commit in the beginning of period 1 to an incentive scheme for period 2. If she learns the agent's productivity in the first period she will offer an incentive scheme in the second period which doesn't give any information rent to the agent. The agent could only be induced to reveal his productivity if the principal would offer the second period's information rent already in the first period. But then the low productivity agent could mimic the good type, take the money, and refuse to work (taking his outside option) in the second period. Thus, if the discount factor of the agent is not too small, there will be "substantial pooling" in all equilibria of this game.[12)]

Our model extends this idea to a repeated bargaining model with an arbitrarily long horizon. The intuition is very much the same. A low cost seller has a strong incentive not to reveal his costs too early because he won't be offered more than his costs thereafter. To compensate him for the foregone information rent the buyer would have to offer a very high price in the beginning ($p > 1$). However, this offer would also be accepted by the high cost types who can just refuse to sell for the low price offers thereafter. The crucial point is that once a seller has revealed his type he

[12)] Even in the two period case the structure of the equilibria of this game with a continuum of types is very complicated. See Laffont and Tirole (1988) for a characterization.

doesn't get any information rent thereafter. Together with the monotonicity of the seller's preferences in both his costs and the price offers this implies for our model that there has to be complete pooling in all but the last periods.

2.6.2. Information Transmission with Two Types

In the end of the game, however, there is at least some transmission of information. It is possible to show what happens in the end of the game for the most simple case with only two different types of the seller and no discounting. Since we show existence in this case by constructing the equilibrium we can drop the assumption of a finite price grid and set $P = \mathbb{R}$. Furthermore, in this case we can show that a seller with costs of 1 will accept p_t if and only if $p_t \geq 1$ in equilibrium. So the behaviour of the commitment type is determined endogenously. The (generically) unique equilibrium path of this case is given in Proposition 2.3.

Proposition 2.3: Let $c \in \{0, 1\}$, $\delta = \beta = 1$, $b > 1$ and $\mu^* = \mu > 0$. Consider the generic case where there is no integer m, such that $\mu = \frac{1}{b^m}$. Then there exists a sequential equilibrium and every SE has the following "on the equilibrium path" strategies and value functions:

Strategy of S with $c = 1$: Accept p_t iff $p_t \geq 1$.

Strategy of S with $c = 0$: If $p_t < 1$ reject p_t with probability $x_t = min\left\{\frac{\mu_t}{1-\mu_t} \cdot (b^{t-1} - 1),\ 1\right\}$, otherwise accept.

Strategy of B: Offer $p_t = 0$ if $\mu_t \in [0, \frac{1}{b^t})$ and $p_t = 1$ if $\mu_t \in (\frac{1}{b^t}, 1]$. If $\mu_t = \frac{1}{b^t}$ offer $p_t = 1$ with probability p_{t+1} and $p_t = 0$ with probability $(1 - p_{t+1})$.

$$V_t^S(c = 1) \quad = \quad 0$$

$$V_t^S(c = 0) \quad = \quad \begin{cases} t - n & \text{if } \exists\ n \text{, such that } \frac{1}{b^n} \geq \mu_t > \frac{1}{b^{n+1}} \text{ and } n < t \\ p_{t+1} & \text{if } \exists\ n \text{, such that } \frac{1}{b^n} = \mu_t \text{ and } n = t \\ 0 & \text{if } \exists\ n \text{, such that } \frac{1}{b^n} > \mu_t \geq \frac{1}{b^{n+1}} \text{ and } n \geq t \end{cases}$$

$$V_t^B = \begin{cases} t \cdot b - (t-n) - \mu_t \cdot \sum_{j=1}^{t} b^j & \text{if } \exists\ n, \text{ such that } \frac{1}{b^n} \geq \mu_t > \frac{1}{b^{n+1}} \text{ and } n < t \\ t \cdot b - \mu_t \cdot \sum_{j=1}^{t} b^j & \text{if } \exists\ n, \text{ such that } \frac{1}{b^n} \geq \mu_t > \frac{1}{b^{n+1}} \text{ and } n \geq t \end{cases}$$

The proof of the proposition is by induction and is left to the reader. Note that the equilibrium strategies satisfy the WMP. However, the equilibrium is not strong Markov because the buyer's action depends on his last period's price offer if $\mu_t = \frac{1}{b^t}$.

Figure 2.1: Evolution of beliefs along the equilibrium path.

The pattern of the equilibrium path is familiar from the equilibrium path of the chain store game analyzed by Kreps and Wilson (1982a). It is graphically illustrated in Figure 2.1. In the beginning of the game, i.e. as long as $\frac{1}{b^t} < \mu$, the buyer always offers 1, which is accepted by both types of sellers, and her probability assessment remains unchanged. If $t = n$, where n is the biggest integer such that $\mu < \frac{1}{b^n}$ the buyer starts offering 0, which is rejected for sure by $c = 1$. The seller with type $c = 0$ randomizes. If he accepts $p_n = 0$ he has revealed his type and will accept $p_{t-j} = 0$ from then on with probability 1. If he rejects p_n the buyer updates her probability assessment that $c = 1$ such that she is just indifferent between offering $p_{n-1} = 0$ and $p_{n-1} = 1$. However, in equilibrium she will offer $p_{n-1} = 0$ with probability 1. Otherwise the seller would have strictly preferred to reject $p_n = 0$. In period $n - 1$ the seller with costs of 0 will again randomize, but he now accepts with a higher probability, and so on. Only in the last period the seller with $c = 0$ will accept $p_1 = 0$ with probability 1 given that he has rejected all other price offers smaller than 1 before.

Note that the expected payoff of the low cost seller after period n is 0. Nevertheless he has to randomize between accepting and rejecting in order to prevent the buyer from offering a price of 0 already before period n. "Reputation building", in the sense that μ_t increases, might only occur after period n when the expected payoff of the seller is 0 anyway. Thus, the seller does not actually gain from building up a reputation. It is also misleading to say that he will "cash in" his reputation when he finally accepts $p_{n-j} = 0$ because this just gives a payoff of 0. The important point is that if the seller were offered prices smaller than 1 before period n then it would pay for him to build up a reputation by rejecting them. Thus it is only the possibility to build up a reputation which prevents the buyer from testing his type.

2.6.3. No Successive Skimming

If there are more than two different types of the seller the analysis of the end of the game is much more complex. However, we can get some insights by comparing the repeated bargaining game with a single trade bargaining game with repeated offers as analyzed by Fudenberg, Levine and Tirole (1985) or Gul, Sonnenschein and Wilson (1986) and the literature on the Coase conjecture. In these games an uninformed seller makes a sequence of take-it-or-leave-it price offers to an informed buyer whose reservation value v is drawn from an interval $[\underline{v}, \overline{v}]$ according to a strictly positive density function. Once an offer has been accepted trade takes place and the game ends, while in our repeated bargaining game there may be trade in every period. A common feature of all single trade bargaining games (STBG) is the "successive skimming property".[13)] To compare the two models consider a modified version of G^T, denoted by $\tilde{G}^T$, in which c is drawn out of $[0,1]$ according to $F(c)$, which is assumed to have a strictly positive density, and in which $P = \mathbb{R}$. In the framework of $\tilde{G}^T$ the successive skimming property can be defined as follows:

> **Definition:** A sequential equilibrium of $\tilde{G}^T$ satisfies the "Successive Skimming Property" (SSP) if after any history h_t and for any price offer p_t there is a cut-off type $\tilde{c}(p_t, h_t)$, such that p_t is accepted if and only if $c \leq \tilde{c}(p_t, h_t)$.

Successive skimming implies that every type of the seller plays according to a pure strategy in equilibrium. Furthermore, information is transmitted gradually in the sense that the higher the costs of the seller the later he will accept a price offer smaller than 1. The SSP very much facilitates the updating of the beliefs (a tail of the density function is cut off after every period) and the construction of an equilibrium. In fact this is always the first property which is shown to hold in all STBG. However, in a repeated bargaining game this property does not hold as is shown in Proposition 2.4.

> **Proposition 2.4:** If $T \geq 3$ and if the discount factor of the seller is not too small ($\delta \geq 0.62$) then there is no sequential equilibrium of $\tilde{G}^T$ satisfying the Successive Skimming Property except for a trivial case.[14)]

The proof is given in the appendix. The fact that the SSP does not hold in a repeated bargaining game is the main analytical difference to single trade bargaining games with repeated offers and the main reason why we cannot say more about information

---

13) See Fudenberg, Levine and Tirole (1985), Lemma 2.1.

14) A similar result has been shown independently for an example by Fernandez and Kofman (1990). They consider the case of a long-run player facing a sequence of short-run players and a uniform distribution of types.

transmission in the end of G^T. Of course, the equilibrium outcomes are different as well. If the lowest possible valuation of the buyer strictly exceeds the costs of the seller in the STBG, then there is a (generically) unique sequential equilibrium with some price discrimination in the beginning of the game, but prices quickly converge to $\underline{v}$. The speed of convergence and the expected payoffs of the players depend crucially on the discount factors. In the repeated bargaining game, however, price discrimination may only occur in the end of the game. This is why discounting plays a much weaker role here.

2.6.4. Two-Sided Uncertainty

An interesting extension of the model would be to allow for two-sided uncertainty, e.g. that the valuation of the buyer is private information as well. As long as the lowest possible reservation value of the buyer strictly exceeds the highest possible costs of the seller the results of this chapter still hold. If the possible valuations overlap, however, the line of reasoning developed here does no longer go through. The reason is that if the buyer has a reservation value which is smaller than the highest possible costs of the seller she cannot guarantee herself a strictly positive payoff which is a necessary ingredient of Lemma 2.4. Furthermore it may no longer be optimal for a low cost seller to mimic the type with highest possible costs which is crucial for Proposition 2.2. Therefore we have to leave the analysis of this case to future research.

2.7. Appendix

Lemma 2.4: Assume $\mu^* > 0$. Fix an integer M,

$$M \;\geq\; N \;=\; \frac{ln(1-\beta) + ln(b-1) - ln\, b}{ln\, \beta} \;>\; 0, \tag{2.9}$$

and define a positive number ϵ,

$$\epsilon \;=\; \frac{(1-\beta)\cdot(b-1)}{b} - \beta^M \;>\; 0\;, \tag{2.10}$$

Consider a history h_t of any equilibrium in which all price offers p_j $(j = T, \cdots, t)$ have been accepted if and only if $p_j \geq 1$. If there have been M price offers $p_j < 1$ along such a history, at least one of them must have had a probability of acceptance of at least ϵ.

<u>Proof:</u> Consider any equilibrium and fix a history h_t in which the seller has accepted p_{t+j} iff $p_{t+j} \geq 1$ $(j = 1, \cdots, T-t)$. Because of $\mu^* > 0$ such a history exists with prositive probability. Note that ϵ is independent of T and t and that M has been chosen in a way to guarantee that $\epsilon > 0$. Consider a possibly mixed strategy of the buyer which chooses prices lower than 1 along h_t in at least M periods with positive probability and suppose that the probability of acceptance for each of the first M of these offers is smaller than ϵ. We will show that this can't be true in equilibrium, because then the buyer would get less than the lower bound of his equilibrium payoff.

Call the first M periods in which price offers lower than 1 are offered with positive probability $\tau_1, \tau_2, \cdots, \tau_M$. Denote by $V^B_{\tau-1}(p_\tau, A)$ the expected equilibrium payoff from the beginning of period $\tau - 1$ onwards for the rest of the game, given the history up to but not including period τ and given that in period τ the price p_τ was accepted (R if p_τ was rejected). In period τ_1, when the first price $p_{\tau_1} < 1$ is offered, the expected payoff for the rest of the game is given by:

$$
\begin{aligned}
V^B_{\tau_1}(p_{\tau_1} < 1) \;=\;& \pi_{\tau_1}(p_{\tau_1}) \cdot \big(b - p_{\tau_1} + \beta \cdot V^B_{\tau_1-1}(p_{\tau_1}, A)\big) \\
&+\; (1 - \pi_{\tau_1}(p_{\tau_1})) \cdot \left(0 + \sum_{j=1}^{\tau_1-\tau_2-1} \beta^j \cdot (b-1)\right) \\
&+\; (1 - \pi_{\tau_1}(p_{\tau_1})) \cdot \pi_{\tau_2}(p_{\tau_2}) \cdot \beta^{\tau_1-\tau_2} \cdot \big(b - p_{\tau_2} + \beta \cdot V^B_{\tau_2-1}(p_{\tau_2}, A)\big) \\
&+\; \cdots + \\
&+\; \prod_{j=1}^{M-1} \big(1 - \pi_{\tau_j}(p_{\tau_j})\big) \cdot \sum_{j=\tau_1-\tau_{(M-1)}}^{\tau_1-\tau_M-1} \beta^j \cdot (b-1) \\
&+\; \prod_{j=1}^{M-1} \big(1 - \pi_{\tau_j}(p_{\tau_j})\big) \cdot \pi_{\tau_M}(p_{\tau_M}) \cdot \beta^{\tau_1-\tau_M} \big(b - p_{\tau_M} + \beta \cdot V^B_{\tau_M-1}(p_{\tau_M}, A)\big) \\
&+\; \prod_{j=1}^{M} \big(1 - \pi_{\tau_j}(p_{\tau_j})\big) \cdot \big(0 + \beta^{\tau_1-\tau_M+1} \cdot V^B_{\tau_M-1}(p_{\tau_M}, R)\big)\; .
\end{aligned}
\tag{2.29}
$$

Note that for all j, $j = 0, 1, \cdots, M$,

$$
b - p_{\tau_j} + \beta \cdot V^B_{\tau_j-1}(p_{\tau_j}, A) \;\leq\; \sum_{j=0}^{\tau_j-1} \beta^j \cdot b \;<\; \frac{b}{1-\beta} \tag{2.30}
$$

and that

$$
V^B_{\tau_M-1}(p_{\tau_M}, R) \;\leq\; \sum_{j=0}^{\tau_M-2} \beta^j \cdot b \;<\; \frac{b}{1-\beta}. \tag{2.31}
$$

Now suppose that all $\pi_{\tau_j}(p_{\tau_j}) < \epsilon$, $j = 0, 1, \cdots, M$. Furthermore we can use that $(1 - \pi_\tau(p_\tau)) \leq 1$ to get

$$
\begin{aligned}
V^B_{\tau_1}(p_{\tau_1} < 1) \;<\;& \epsilon \cdot \frac{b}{1-\beta} + \sum_{j=1}^{\tau_1-\tau_2-1} \beta^j \cdot (b-1) + \epsilon \cdot \beta^{\tau_1-\tau_2} \cdot \frac{b}{1-\beta} + \cdots + \\
&+ \sum_{j=\tau_1-\tau_{(M-1)}}^{\tau_1-\tau_M-1} \beta^j \cdot (b-1) + \epsilon \cdot \beta^{\tau_1-\tau_M} \cdot \frac{b}{1-\beta} + \beta^{\tau_1-\tau_M+1} \cdot \frac{b}{1-\beta}.
\end{aligned}
\tag{2.32}
$$

Consider the first and the last term of this expression. It is easy to check that ϵ has been chosen in a way that

$$
\epsilon \cdot \frac{b}{1-\beta} + \beta^{\tau_1-\tau_M+1} \cdot \frac{b}{1-\beta} \;\leq\; b - 1. \tag{2.33}
$$

Given this it is certainly true that $\beta^{\tau_1-\tau_j} \cdot \epsilon \cdot \frac{b}{1-\beta} < \beta^{\tau_1-\tau_j} \cdot (b-1)$. Therefore we have

$$\begin{aligned} V^B_{\tau_1}(p_{\tau_1} < 1) \; < \; & b - 1 + \sum_{j=1}^{\tau_1-\tau_2-1} \beta^j \cdot (b-1) + \beta^{\tau_1-\tau_2} \cdot (b-1) + \cdots + \\ & + \sum_{j=\tau_1-\tau_{(M-1)}}^{\tau_1-\tau_M-1} \beta^j \cdot (b-1) + \beta^{\tau_1-\tau_M} \cdot (b-1) \qquad (2.34) \\ = & \sum_{j=0}^{\tau_1-\tau_M} \beta^j \cdot (b-1) \; \leq \; \sum_{j=0}^{\tau_1-1} \beta^j \cdot (b-1). \end{aligned}$$

But this is a contractiction to the fact that we are in equilibrium. By Lemma 2.3 we know that the right hand side of the inequality is a lower bound for the equilibrium payoff of the buyer. Therefore, if there are M price offers lower than 1, then at least one of them has a probability of acceptance bigger than ϵ. *Q.E.D.*

Proposition 2.4: If $T \geq 3$ and if the discount factor of the seller is not too small ($\delta \geq 0.62$) then there is no sequential equilibrium of $\tilde{G}^T$ satisfying the Successive Skimming Property except for a trivial case.

Proof: Suppose there is an equilibrium in which the SSP does hold. Then after any history up to but not including period t the support of the buyer's beliefs is given by an intervall $\left[\underline{c}(h_t), \overline{c}(h_t)\right]$, $0 \leq \underline{c}(h_t) < \overline{c}(h_t) \leq 1$. The reference to history is omitted from now on. Take any history h_3 and suppose that any $p_3 < \overline{c}$ has been offered in period 3, which need not have been an equilibrium price offer. Three cases have to be distinguished:

(1) $\tilde{c}(p_3) = \overline{c}$, i.e. every seller with $c \leq \overline{c}$ accepts p_3. Consider the seller with $c = \overline{c}$. Since $p_3 < \overline{c}$ he loses $p_3 - \overline{c}$ in period 3 for which he cannot be compensated in the future, because no $p > \overline{c}$ will be offered in periods 2 or 1 in any equilibrium of the following subform.[15] Hence he would have done better rejecting, a contradiction.

[15] This is easy to see if there are only two periods left. However, if there are more

(2) $\underline{c} \leq \tilde{c}(p_3) < \overline{c}$. For type $\tilde{c}(p_3)$ it must be true that

$$p_3 - \tilde{c}(p_3) \geq \delta \cdot [E(p_2^R) - \tilde{c}(p_3)] + \delta^2 \cdot [E(p_1^{RA}) - \tilde{c}(p_3)], \tag{2.35}$$

or equivalently that

$$(\delta^2 + \delta - 1) \cdot \tilde{c}(p_3) \geq \delta E(p_2^R) + \delta^2 E(p_2^{RA}) - p_3. \tag{2.36}$$

Note that after p_3 has been accepted the seller believes with probability 1 that $c \leq \tilde{c}(p_3)$. Thus he won't offer more than $\tilde{c}(p_3)$ in the future and the payoff of type $\tilde{c}(p_3)$ after accepting is $p_3 - \tilde{c}(p_3)$. On the other hand if the seller with type $\tilde{c}(p_3)$ would have rejected p_3 he certainly will accept both subsequent price offers, p_2^R and p_1^{RA}, in any equilibrium of the following subform. This is true because it can't be an equilibrium that p_2 or p_1 are rejected with probability 1. But $\tilde{c}(p_3)$ is the smallest possible type in this subform. Thus, given the SSP, if there is anybody who accepts p_2 or p_1 at all then $\tilde{c}(p_3)$ has to accept them as well.

Now consider a seller with slightly higher costs $c' = \tilde{c}(p_3) + \epsilon > \tilde{c}(p_3)$. If c' is close enough to $\tilde{c}(p_3)$ he will accept p_2^R and p_1^{RA} by the same argument as type $\tilde{c}(p_3)$. However, for him it must be true that

$$p_3 - c' < \delta \cdot [E(p_2^R) - c'] + \delta^2 \cdot [E(p_1^{RA}) - c'], \tag{2.37}$$

or equivalently that

$$(\delta^2 + \delta - 1) \cdot c' < \delta E(p_2^R) + \delta^2 E(p_1^{RA}) - p_3. \tag{2.38}$$

Combining (2.36) and (2.38) yields

$$(\delta^2 + \delta - 1) \cdot c' < \delta E(p_2^R) + \delta^2 E(p_1^{RA}) - p_3 \leq (\delta^2 + \delta - 1) \cdot \tilde{c}(p_3). \tag{2.39}$$

than two periods left it might happen that there are some price offers $p > \overline{c}$ in the future. To exclude this problem we would have to impose the Weak Markov Property. However, Proposition 2.4 holds for all sequential equilibria. This is why the proof has been built around period 3.

But this is a contradiction to $c' > \tilde{c}(p_3)$ if $\delta \geq 0.62$.

(3) $\tilde{c}(p_3) < \underline{c}$, i.e. every buyer strictly prefers to reject p_3. We have shown that cases (1) and (2) lead to a contradiction for any $p_3 < 1$, but case (3) may of course hold for some p_3. If p_3 is low enough nobody will accept it. However, the SSP has to hold for all price offers p_3. Therefore it is sufficient to find a p_3 for which the SPP is violated. Two subcases have to be distinguished:

(3a) The buyer puts so much probability on the high cost types that she would not even try to price discriminate in the one-shot game. In this case she will offer $p_2 = \overline{c}$ and $p_1 = \overline{c}$ after p_3 has been rejected and nobody will accept p_3. In this trivial case the SSP does of course hold.

(3b) Now suppose that given her probability assessment in the beginning of period 3 the buyer would price discriminate in the one-shot game and offer $p_1^* < \overline{c}$ in the last period. Suppose that in period 3 a price $p_3 = \overline{c} - \epsilon$ is offered and consider the seller with type $c = p_1^*$. If he rejects, the best he can hope for is to get $p_2 = \overline{c}$ and $p_1 = p_1^*$, so his overall payoff is at most $V_R^S(c = p_1^*) = \delta \cdot (\overline{c} - c) + \delta^2 \cdot (p_1^* - c) = \delta \cdot (\overline{c} - p_1^*)$. If he accepts p_3 his payoff is at least $V_A^S(c = p_1^*) = p_3 - c = \overline{c} - \epsilon - p_1^*$. For every $\delta < 1$ there exists an $\epsilon > 0$ such that $\overline{c} - \epsilon - p_1^* > \delta \cdot (\overline{c} - p_1^*)$. Hence, there exists a type $c \geq \overline{c}$ who strictly prefers to accept p_3, a contradiction to $\tilde{c}(p_3) < \underline{c}$.

Therefore, the SSP can only hold in the trivial case of (3a) where there is no price discrimination at all throughout the game. *Q.E.D.*

Chapter 3:

Reputation and Equilibrium Characterization in Repeated Games of Conflicting Interests

3.1. Introduction

In this chapter we take a more general approach to two long-run player games. Consider a repeated relationship between two long-run players one of whom is privately informated about his type. A common intuition is that the informed player may take advantage of the uncertainty of his opponent and enforce an outcome more favourable to him than what he would have got under complete information. This intuition has been called "reputation effect" and has been given considerable attention in the literature. The purpose of this chapter is to formalize this intuition in a general model of repeated games with "conflicting interests". Furthermore we show that the effect is robust against perturbations of the information structure of the game. Fudenberg and Levine (1989) reached a similar conclusion for games in which a long-run player with private information about his type faces a sequence of short-run opponents, each of whom plays only once but observes all previous play. This chapter provides a generalization and qualification of their results for the two long-run player case.

The philosophy of this approach and the related literature have been discussed extensively in Chapter 1. There we argued that if the reputation effect is to have any meaning then it should be robust against small perturbations of the information structure of the game. Surprisingly, insisting on robustness yields a result which is in striking contrast to the message of the Folk-theorem for games with incomplete information. No matter what types may possibly be drawn by nature (including those considered by Fudenberg and Maskin (1986)) and how likely they are to occur, if player one is sufficiently patient, if the game is of "conflicting interests", and if there is an arbitrarily small but positive probability of a "commitment" type then we can give a clearcut prediction of the equilibrium outcome of all Nash equilibria.

To make this more precise, consider a repeated simultaneous move game with complete information in which player one would like to commit himself always to take an action s_1^* called his "commitment action". If player two responds optimally to s_1^* player one gets his "commitment payoff". Assume that the game is of "conflicting interests" in the sense that playing s_1^* holds player two down to her minimax payoff. Now suppose that the information structure of this game is perturbed such that player one may be one of several possible "types" which have different payoff functions. Consider a type for whom it is a dominant strategy in the repeated game always to play s_1^* and call him the "commitment type". Our main theorem says that if the commitment type has any arbitrarily small but positive probability and if player one's discount factor goes to 1, then his payoff in any Nash equilibrium is bounded below by his commitment payoff. This result is independent of the nature of the other possible types and their respective probabilities. Furthermore we show that a necessary and sufficient condition for this generalization to hold is that the game is of conflicting interests.

A complementary analysis to ours is the work of Aumann and Sorin (1989) who consider a different class of repeated games, coordination games with "common interests". However they have to restrict the possible perturbations in the following way. With some probability each of the players may act like any automaton with bounded recall. Aumann and Sorin show that if all strategies of recall zero exist with positive probability then all pure strategy equilibria will be close to the cooperative outcome. Games of "common" and of "conflicting" interests are two polar cases. We will explore these classes of games in more detail in Chapter 3.5.

In the next section we introduce the model following closely Fudenberg-Levine (1989) and summarize briefly their main results. Then we give a counterexample showing that their theorem cannot carry over to the class of all repeated games with two long-run players. This gives some intuition on how this class has to be restricted. Section 3.4 contains our main results and generalizes Fudenberg-Levine's (1989) Theorem 1 for the two long-run player case. We also show that the restriction to games with "conflicting interests" is a necessary condition for our result to hold. Furthermore we extend the analysis to the case of two-sided incomplete information. In Section 3.5 we give several examples which show how restrictive the "conflicting interests" condition is. Here we also discuss the different approach taken in the last chapter. Section 3.6 concludes and briefly outlines several extensions of the model.

3.2. Description of the Game

In most of this chapter we consider the following very simple model of a repeated game which is an adaptation of Fudenberg-Levine (1989) and Fudenberg-Kreps-Maskin (1990) to the two long-run player case. The two players are called "one" (he) and "two" (she). In every period they move simultaneously and choose an action s_i out of their respective action sets S_i, $i \in \{1, 2\}$. Here we will assume that the S_i are finite sets.[1] As a point of reference let us describe the unperturbed game (with complete information) first. $g_i(s_1, s_2)$ is the payoff function of player i in the unperturbed stage game g depending on the actions taken by both players. Let Σ_i denote the set of all mixed strategies σ_i of player i and (in an abuse of notation) $g_i(\sigma_1, \sigma_2)$ the expected stage game payoffs.

G^T is the T-fold repetition of the stage game g, where T may be finite or

[1] See Section 3.6 for the extension to extensive form stage games, continuous strategy spaces and more than two players.

infinite. We will deal in most of this chapter with the infinite horizon case but all of the results carry over immediately to finitely repeated games if T is large enough. In the repeated game the overall payoff for player i from period t onwards (and including period t) is given by

$$V_i^t = \sum_{\tau=t}^{T} \delta_i^{\tau-t} g_i^\tau \tag{3.1}$$

where δ_i denotes his (her) discount factor ($0 \leq \delta_i < 1$). After each period both players observe the actions that have been taken. They have perfect recall and can condition their play on the entire past history of the game. Let h^t be a specific history of the repeated game out of the set $H^t = (S_1 \times S_2)^t$ of all possible histories up to and including period t. A pure strategy for player i is a sequence of maps $s_i^t : H^{t-1} \to S_i$ and is denoted by $s_i = (s_i^1, s_i^2, \cdots)$. Correspondingly, let $\sigma_i = (\sigma_i^1, \sigma_i^2, \cdots)$ denote a mixed (behavioral) strategy of player i, where $\sigma_i^t : H^{t-1} \to \Sigma_i$. For notational convenience the dependence on history is suppressed if there is no ambiguity.

Let $B : S_1 \mapsto \Sigma_2$ be the best response correspondence of player two in the stage game and define

$$g_1^* = \mathop{max}_{s_1 \in S_1} \mathop{min}_{\sigma_2 \in B(s_1)} g_1(s_1, \sigma_2) \tag{3.2}$$

as the "commitment payoff" of player one. g_1^* is the most player one could guarantee for himself in the stage game if he could commit to any pure strategy $s_1 \in S_1$.[2] Note that the minimum over all $\sigma_2 \in B(s_1)$ has to be taken since player two may be indifferent between several best responses to s_1 in which case she may take the response player one prefers least. Let s_1^* (his "commitment action") satisfy

$$\mathop{min}_{\sigma_2 \in B(s_1^*)} g_1(s_1^*, \sigma_2) = g_1^* . \tag{3.3}$$

[2] The analysis can be extended to the more general case where player one would like to commit himself to a strategy dependend on history. See the remarks in Section 3.6

Furthermore, let $\sigma_2^* \in B(s_1^*)$ denote any strategy of player two which is a best response to s_1^* and define

$$g_2^* = g_2(s_1^*, \sigma_2^*) \ . \tag{3.4}$$

So g_2^* is the most player two could get in the stage game if player one were committed to s_1^*. Suppose $B(s_1^*) \neq \Sigma_2$ (otherwise the game is "trivial" because player one's commitment payoff is his minimax payoff). Then there exists a $\tilde{s}_2 \notin B(s_1^*)$ such that

$$\tilde{g}_2 = g_2(s_1^*, \tilde{s}_2) = \max_{s_2 \notin B(s_1^*)} g_2(s_1^*, s_2) < g_2^* \ . \tag{3.5}$$

Note that the maximum exists because it is taken over the finite set of all (pure) actions $s_2 \notin B(s_1^*)$. So $\tilde{g}_2$ is the maximum player two can get if he does not take an action which is a best response against s_1^*, given that player one takes his commitment action. Finally, define the maximal payoff player two can get at all as

$$\overline{g}_2 = \max_{\sigma_2 \in \Sigma_2} \max_{\sigma_1 \in \Sigma_1} g_2(\sigma_1, \sigma_2) \ . \tag{3.6}$$

Clearly, in the repeated game it must be true that

$$V_2^t \leq \sum_{\tau=t}^{\infty} \delta_2^{\tau-t} \cdot \overline{g}_2 = \frac{\overline{g}_2}{1-\delta_2} = \overline{V}_2^t \ . \tag{3.7}$$

for all t and all $h^{t-1} \in H^{t-1}$.

Consider now a perturbation of this complete information game such that in period 0 (before the first stage game is played) the "type" of player one is drawn by nature out of a countable set $\Omega = (\omega_0, \omega_1, \cdots)$ according to the probability measure μ. Player one's payoff function now depends also on his type, so $g_1 : S_1 \times S_2 \times \Omega \rightarrow \mathbb{R}$. The perturbed game $G^T(\mu)$ is a game with incomplete information in the sense of Harsanyi (1967-68). In the perturbed game a strategy of player one may not only depend on history but also on his type, so $\sigma_1^t : H^{t-1} \times \Omega \rightarrow \Sigma_1$. Two types out of the set Ω are of particular importance:

- ω_0 denotes the "normal" type of player one whose payoff function is the same as in the unperturbed game:

$$g_1(s_1, s_2, \omega_0) \;=\; g_1(s_1, s_2). \tag{3.8}$$

In many applications $\mu(\omega_0)$ will be close to 1. However, we only have to require that $\mu(\omega_0) = \mu^0 > 0$.

- ω^* is the "commitment" type for whom it is a dominant strategy in the repeated game always to play s_1^*. This is for example the case if his payoff function satisfies

$$g_1(s_1^*, s_2, \omega^*) \;=\; g_1(s_1^*, s_2', \omega^*) \;>\; g_1(s_1, s_2', \omega^*) \tag{3.9}$$

for all $s_1 \neq s_1^*$ and all $s_2, s_2' \in S_2$. The dominant strategy property in the repeated game implies that in any Nash equilibrium player one of type ω^* has to play s_1^* in every period. This in turn implies that if $\mu(\omega^*) = \mu^* > 0$ then with positive probability there exists a history in any Nash equilibrium with $s_1^t = s_1^*$ for all t. The set of all such histories is denoted by H^*.

We will now restate an important lemma of Fudenberg-Levine (1989) about statistical inference which is basic to the following analysis. The lemma says that if type ω^* has positive probability and if player two observes s_1^* being played in every period then there is a fixed finite upper bound on the number of periods in which player two will believe s_1^* is "unlikely" to be played. The intuition for this result is the following. Consider any history $h^{t-1} \in H^*$ in which player one has always played s_1^* up to period $t-1$. Suppose player two believes that the probability of s_1^* being played in period t is smaller than $\overline{\pi}$, $0 \leq \overline{\pi} < 1$. If player two observes s_1^* being played in t she is "surprised" to some extent and will update her beliefs. Because the commitment type chooses s_1^* with probability 1 while player two expected s_1^* to be played with a probability bounded away from 1 it follows from Bayes' law that the updated probability that she faces the commitment type has to increase by an amount bounded away from 0. However, this cannot happen arbitrarily often because the updated probability of the commitment type cannot become bigger than 1. This gives the upper bound on the number of periods in which player two may expect s_1^* to be played with a probability less than $\overline{\pi}$. Note that this argument is independent of the discount factors of the two players.

To put it more formally: Each (possibly mixed) strategy profile (σ_1, σ_2) induces a probability distribution π over $(S_1 \times S_2)^\infty \times \Omega$. Given a history h^{t-1} let $\pi^t(s_1^*)$ be the probability attached by player two to the event that the commitment strategy is being played in period t, i.e. $\pi^t(s_1^*) = Prob(s_1^t = s_1^* \mid h^{t-1})$. Note that since h^{t-1} is a random variable $\pi^t(s_1^*)$ is a random variable as well. Fix any $\overline{\pi}$, $0 \le \overline{\pi} < 1$, and consider any history h induced by (σ_1, σ_2). Along this history h let $n(\overline{\pi}, h)$ be the number (possibly infinite) of the random variables $\pi^t(s_1^*)$ for which $\pi^t(s_1^*) \le \overline{\pi}$. Again, since h is a random variable, so is $n(\overline{\pi}, h)$.

Lemma 3.1: Let $0 \le \overline{\pi} < 1$. Suppose $\mu(\omega^*) = \mu^* > 0$, and that (σ_1, σ_2) are such that $Prob(h \in H^* \mid \omega^*) = 1$. Then

$$Prob\left[n(\overline{\pi}, h) > \frac{log\,\mu^*}{log\,\overline{\pi}} \mid h \in H^*\right] \;=\; 0. \qquad (3.10)$$

Furthermore, for any infinite history h such that the truncated histories h_t all have positive probability and such that s_1^* is always played, $\mu(\omega^* \mid h_t)$ is nondecreasing in t.

Proof: See Fudenberg-Levine (1989), Lemma 1.

One feasible strategy for player one with type ω_0 is of course to mimic the commitment type and always to play s_1^*. Lemma 3.1 does not say that in this case $\mu(\omega^* \mid h_t \in h^*)$ converges to 1, i.e. that player two will gradually become convinced that she is facing ω^* if she observes s_1^* always being played. Rather it says that if she observes s_1^* being played in every period she cannot believe arbitrarily often that s_1^* is "unlikely" to be played.

Suppose that player two is completely myopic, that is she is only interested in her payoff of the current period. Fudenberg and Levine show that there is a $\overline{\pi} < 1$ such that if the probability that player one will play s_1^* is bigger than $\overline{\pi}$ then a short-run player two will choose a best response against s_1^*. Thus, if player one mimics the commitment type, then by Lemma 3.1 his short-run opponents will take $s_2 \notin B(s_1^*)$ in at most $k = \frac{log\,\mu^*}{log\,\overline{\pi}}$ periods. The worst that can happen to player one is that these k periods occur in the beginning of the game and that in each of these periods he gets

$$\underline{g}_1 = \min_{\sigma_2 \in \Sigma_2} g_1(s_1^*, \sigma_2). \tag{3.11}$$

This argument provides the intuition for the following theorem.

Theorem 3.1: Let $\delta_2 = 0$, $\mu(\omega^0) > 0$, and $\mu(\omega^*) = \mu^* > 0$. Then there is a constant $k(\mu^*)$ otherwise indepedent of (Ω, μ), such that

$$V_1(\delta_1, \mu^*; \omega^0) \;\geq\; \sum_{t=1}^{k(\mu^*)} \delta_1^{t-1} \underline{g}_1 \;+\; \sum_{t=k(\mu^*)+1}^{\infty} \delta_1^{t-1} g_1^* \;, \tag{3.12}$$

where $V_1(\delta_1, \mu^*; \omega^0)$ is any equilibrium payoff of player one with type ω_0 in any Nash equilibrium of $G^\infty(\mu)$.

Proof: See Fudenberg-Levine (1989), Theorem 1.

If δ_1 goes to 1 the "normal" type of player one can guarantee himself on average at least his commitment payoff no matter what other types may be around with positive probabilty. The result is discussed in more detail in Fudenberg-Levine (1989). Note however that Theorem 3.1 is crucially based on the assumption that player two is completely myopic. If she cares about future payoffs then she may trade off short-run losses against long-run gains. Thus, even if she believes that s_1^* will be played with a probability arbitrarily close or equal to 1, she may take an action s_2 which is not a short-run best response against s_1^*. One intuitive reason for this could be that she might invest in screening the different types of player one.

Even if this yields losses in the beginning of the game the investment may well pay off in the future. This leads Fudenberg and Levine to conclude that their result does not apply to two long-run player games. The main point of our analysis, however, is to show that for a more restricted class of games a similar result holds in the two long-run player case as well. Since player two's discount factor is smaller than 1 the returns from an investment may not be delayed to far to the future. She will not "test" player one's type arbitrarily often if the probability that he will play s_1^* is always arbitrarily close to one. This idea will be used in Section 3.4 to prove an analogue of Theorem 3.1 for two long-run player games.

3.3. A Game with no Conflicting Interests

Before establishing our main result let us show that Theorem 3.1 cannot carry over to all repeated games with two long-run players. We give a counterexample in which the normal type of player one cannot guarantee himself almost his commitment payoff in all Nash equilibria of the repeated game. The example is instructive because it leads to a necessary and sufficient condition for the generalization to hold. Consider an infinite repetition of the following stage game with 3 types of player one:

	L	R
U	10, 10	0, 0
D	0, 0	1, 1

"normal" type
$\mu^0 = 0.8$

	L	R
U	10, 10	10, 0
D	0, 0	1, 1

"commitment" type
$\mu^* = 0.1$

	L	R
U	1, 10	1, 0
D	1, 0	1, 1

"indifferent" type
$\mu^i = 0.1$

Figure 3.1: A game with common interests

Player one chooses between U and D and his payoff is given in the upper left corners of each cell. Clearly the normal type of player one would like to publicly commit always to play U which would give him a commitment payoff of 10 per period in every Nash equilibrium. For the commitment type it is indeed a dominant strategy in the repeated game always to play U. The indifferent type, however, is indifferent between U and D no matter what player two does.

Consider the following set of strategies and beliefs:

- Strategy of the *normal type of player one*: "Always play U. Only if you ever played D, switch to always playing D."
- Strategy of the *commitment type of player one*: "Always play U."
- Strategy of the *indifferent type of player one*: "Always play U along the equilibrium path. If there has been any deviation by any player in the past switch to always playing D."
- Strategy of *player two*: "Alternate playing 19 times L and 1 times R along the equilibrium path. If player one ever played D, go on playing R forever. If player two herself deviated in the last period, play L in the following period. If player one reacted to the deviation by playing U, go on playing L forever. If he reacted with D, play R forever."
- *Beliefs*: Along the equilibrium path beliefs don't change. If player two ever observes D to be played she puts probability 0 on the commitment type. If player one reacts to a deviation of player two by playing U the indifferent type gets probability 0. In both cases the respective two other types may get arbitrary probabilities which add up to 1.

It is easy to check that this is a sequential equilibrium of G^{∞} if $0.75 \leq \delta_1$ and $0.95 \leq \delta_2$.

Note that there are very few restrictions imposed on the updating of beliefs in information sets which are not reached on the equilibrium path. The example only requires that if D is played for the first time the commitment type gets probability 0, which is perfectly reasonable given that for him it is a dominant strategy in the repeated game always to play U. The problem for the normal type is that he can only signal his type by playing D which eliminates the commitment type. However, without the commitment type there is no possibility to get rid of the "bad" equilibrium (D, R) in the subform after player one deviated to D. Note further that although player two is sure that her opponent will play the commitment action (U) with probability 1 she plays R (which is not a short-run best response) in one of twenty periods. This is even more surprising given that player two would like to face an opponent who is committed to play the commitment action because this would give her an average payoff of 10 as well. The problem for player two is that it might happen that her short-run best response triggers with some probability a continuation equilibrium which gives her a payoff far worse than what she would have got from playing against the commitment type or against someone who is eager to mimic the commitment type. This risk may sustain all kind of equilibrium outcomes in the two long-run player case.

In this equilibrium the normal type of player one gets an average payoff of 9.5 if δ_1 goes to 1 instead of the commitment payoff of 10.[3)] So the result of Fudenberg and Levine does not carry over to this example. However, in the next section we show that there is an interesting class of repeated games - games with *conflicting interests* - in which it is possible to generalize Theorem 3.1 for the two long-run player case.

[3)] To keep the example as simple as possible we have put a rather high prior probability on the commitment and on the indifferent type. Proposition 3.1 in the next section shows that this is not necessary. Even if the prior probability of the normal type of player one is arbitrarily close to one it is possible to construct a sequential equilibrium in which his payoff is strictly below his commitment payoff if δ_1 goes to 1.

3.4. Main Results

3.4.1. The Theorem

Suppose that the commitment strategy of player one holds player two down to her minimax payoff. In this case there is no "risk" in playing a best response against s_1^* because player two cannot get less than her minimax payoff in any continuation equilibrium.

> **Definition:** A game g is called a game of "conflicting interests" if the commitment strategy of player one holds player two down to her minimax payoff, i.e. if
>
> $$g_2^* \ = \ g_2(s_1^*, \sigma_2^*) \ = \ \underset{\sigma_1}{min}\, \underset{\sigma_2}{max}\, g_2(\sigma_1, \sigma_2). \tag{3.13}$$

"Conflicting interests" are a necessary and sufficient condition for our main result. Note that the definition puts no restriction on the possible perturbations of the payoffs of player one. It is a restriction only on the unperturbed game g, i.e. on the commitment strategy and on the payoff function of player two. We will discuss this class of games extensively and give several examples in Section 5. Clearly, in a game with conflicting interests player two can guarantee herself in any continuation equilibrium after any history h_t at least

$$\underline{V}_2 \ = \ \frac{1}{1 - \delta_2} \cdot g_2^* \ . \tag{3.14}$$

This lower bound is crucial to establish the following lemma:

> **Lemma 3.2:** Let g be a game of conflicting interests and let $\mu(\omega^*) = \mu^* > 0$. Consider any Nash equilibrium $(\hat{\sigma}_1, \hat{\sigma}_2)$ and any history h^* consistent with this equilibrium in which player one always plays s_1^*. Suppose that,

given this history up to period t, the equilibrium strategy of player two prescribes to take $s_2^{t+1} \notin B(s_1^*)$ with positive probability in period $t+1$. For any δ_2, $0 < \delta_2 < 1$, there exists an finite integer M,

$$M \;\geq\; N \;=\; \frac{ln(1-\delta_2) + ln(g_2^* - \tilde{g}_2) - ln(\overline{g}_2 - \tilde{g}_2)}{ln\,\delta_2} \;>\; 0, \tag{3.15}$$

and a positive ϵ,

$$\epsilon \;=\; \frac{(1-\delta_2)^2 \cdot (g_2^* - \tilde{g}_2)}{\overline{g}_2 - \tilde{g}_2} - \delta_2^M \cdot (1-\delta_2) \;>\; 0\;, \tag{3.16}$$

such that in at least one of the periods $t+1, t+2, \cdots, t+M$ the probability that player one does not play s_1^* must be at least ϵ.

Proof: See Appendix.

Let us briefly outline the intuition behind this result. Since g is of conflicting interests player two can guarantee herself at least $\underline{V}_2^{t+1} = \frac{g_2^*}{1-\delta_2}$ in any continuation equilibrium after any history h_t. Therefore, if she tries to test player one's type and takes an action $s_2^{t+1} \notin B(s_1^*)$ in period $t+1$ this must give her an expected payoff of at least $\underline{V}_2^{t+1}$ in the overall game. If player one chooses s_1^* with a probability arbitrarily close or equal to 1, then playing $s_2 \notin B(s_1^*)$ yields a "loss" of at least $g_2^* - \tilde{g}_2 > 0$ in this period. Recall that $\tilde{g}_2$ is defined as the maximal payoff player two gets if she does not take a best response against s_1^*. On the other hand, $\overline{g}_2$ is an upper bound on what player two may get in any period in which player one does not take his commitment action, and - of course - she cannot get more than g_2^* if he plays s_1^*. But if future payoffs are bounded and $\delta_2 < 1$ then the compensation for an expected loss today must not be delayed too far in the future. M and ϵ are constructed such that if player two takes action $s_2^{t+1} \notin B(s_1^*)$ in period $t+1$ then it cannot be true that in each of the next M periods the probability that player one takes his commitment action is bigger than $(1-\epsilon)$. Otherwise player two would get less than her minimax payoff in equilibrium, a contradiction.

Note that Lemma 3.2 holds in any proper subform of G as long as player one always played s_1^* in the history up to that subform. Thus if player two chooses actions $s_2 \notin B(s_1^*)$ along h^* in $n \cdot M$ periods, then in at least n of these periods the probability that player one does not play s_1^* must be at least ϵ. Together with Lemma 3.1 this implies our main theorem:

Theorem 3.2: Let g be of conflicting interests and let $\mu(\omega^0) > 0$ and $\mu(\omega^*) = \mu^* > 0$. Then there is a constant $k(\mu^*, \delta_2)$ otherwise independent of (Ω, μ), such that

$$V_1(\delta_1, \delta_2, \mu^*; \omega^0) \;\geq\; \sum_{t=1}^{k(\mu^*,\delta_2)} \delta_1^{t-1} \underline{g}_1 \;+\; \sum_{t=k(\mu^*,\delta_2)+1}^{\infty} \delta_1^{t-1} g_1^* \;, \tag{3.17}$$

where $V_1(\delta_1, \delta_2, \mu^*; \omega^0)$ is any equilibrium payoff of player one with type ω_0 in any Nash equilibrium of $G^\infty(\mu)$.

<u>Proof:</u> Consider the strategy for the normal type of player one of always playing s_1^*. Take the integer $M = [N] + 1$, where $[N]$ is the biggest integer less than N, and a real number $\epsilon > 0$, where N and ϵ are defined in Lemma 3.2. By Lemma 3.2 we know that if player two takes an action $s_2 \notin B(s_1^*)$ then there is at least one period (call it τ_1) among the next M periods in which the probability that player one will play s_1^* (denoted by $\pi_{\tau_1}^*$) is smaller than $(1 - \epsilon)$. So

$$\pi_{\tau_1}^* \;<\; 1 - \epsilon \;\equiv \overline{\pi} \;. \tag{3.18}$$

However, by Lemma 3.1 we know that

$$\pi \,\left[n(1-\epsilon, h) \;>\; \frac{ln\ \mu^*}{ln\ (1-\epsilon)} \;\;|\;\; h \in H^* \right] \;=\; 0 \;. \tag{3.19}$$

That is, the probability that player one takes his commitment action cannot be smaller than $1 - \epsilon$ in more than $\frac{ln\ \mu^*}{ln\ (1-\epsilon)}$ periods. Therefore, player two cannot choose actions $s_2 \notin B(s_1^*)$ more often than

$$k \;=\; M \cdot \frac{ln\ \mu^*}{ln(1-\epsilon)} \tag{3.20}$$

times. Substituting $M = [N] + 1$ and ϵ from Lemma 3.2, we get

$$k(\mu^*, \delta_2) \;=\; ([N]+1) \cdot \frac{ln\ \mu^*}{ln\left(1 - \frac{(1-\delta_2)\cdot(g_2^* - \tilde{g}_2)}{\bar{g}_2 - \tilde{g}_2} + \delta_2^{[N]+1}\right)}. \tag{3.21}$$

In the worst case player two chooses these actions in the first $k(\mu^*, \delta_2)$ periods. This gives the lower bound of the theorem. *Q.E.D.*

> **Corollary 3.1:** If $\delta_1 \to 1$ (keeping δ_2 fixed) then the limit of the average payoff of the normal type of player one in any Nash equilibrium of the perturbed game is bounded below by his commitment payoff.

Note that the proofs of Lemma 3.2 and Theorem 3.2 immediately carry over to finite repetitions of g and to the case where $\delta_1 = 1$. The same holds for Corollary 3.1 if the number of periods goes to infinity.

Theorem 3.2 and Corollary 3.1 are in striking contrast to the message of the Folk theorem for games with incomplete information by Fudenberg and Maskin (1986). The Folk theorem says that any feasible payoff vector which gives each of the players at least his minimax payoff can be sustained as an equilibrium outcome of the perturbed game if the "right" perturbation has been chosen. Our Theorem 3.2, however, gives a clearcut prediction of the equilibrium outcome in any Nash equilibrium no matter how the game is perturbed (as long as the commitment type has positive probability). If player one is patient enough, he will get his commitment payoff.

Note that Fudenberg and Maskin's Folk theorem assumes that both players do not discount future payoffs, while our result requires that δ_2 is strictly smaller than 1. This is necessary because $k(\mu^*, \delta_2)$ is an increasing function of δ_2 which goes to infinity if δ_2 goes to 1, i.e. the more patient player two is the longer she may take actions which are not a best reply against s_1^*. Recall that δ_2 has been kept fixed in Corollary 3.1. Although player two's discount factor may be arbitrarily close to one, taking the limit as $\delta_1 \to 1$ ensures that player one is sufficiently "more patient". So the relative patience of the players is very important. This is quite natural as will become clear from the discussion of two-sided uncertainty in Section 4.3.

Fudenberg and Levine's Theorem 3.1 can be obtained as a special case of Theorem 3.2 for the class of games with conflicting interests. Note that if δ_2 goes to 0 then N (as defined in Lemma 3.2) goes to 0. That is, if player two chooses an action which is not a best response against s_1^*, then the probability that player one does not play s_1^* must be at least ϵ in this period already. So

$$\lim_{\delta_2 \to 0} k(\mu^*, \delta_2) \;=\; \frac{ln\,\mu^*}{ln\left(1 - \frac{g_2^* - \tilde{g}_2}{\overline{g}_2 - \tilde{g}_2}\right)} \;=\; \frac{ln\,\mu^*}{ln\left(\frac{\overline{g}_2 - g_2^*}{\overline{g}_2 - \tilde{g}_2}\right)} \; . \tag{3.22}$$

In a game with conflicting interests a short-run players two will play a best response against s_1^* if

$$g_2^* \;>\; \pi \cdot \tilde{g}_2 + (1 - \pi) \cdot \overline{g}_2 \tag{3.23}$$

or, equivalently, if

$$\pi \;>\; \frac{\overline{g}_2 - \tilde{g}_2}{\overline{g}_2 - \tilde{g}_2} \;\equiv \overline{\pi} \; . \tag{3.24}$$

Using (23) in Lemma 3.1 immediately implies Theorem 3.1.

Finally, it is worth mentioning that Theorem 3.2 only requires $\mu(\omega^0) > 0$. For example, there may be different "normal" types, each of whom has his own commitment type. If the respective unperturbed game for each of these normal types is of conflicting interests, then each of them will obtain at least his commitment payoff if his discount factor is close enough to 1.

3.4.2. Necessity of the "Conflicting Interests" Condition

The question arises whether Theorem 3.2 also holds for games which are not of conflicting interests. If the game is not "trivial" in the sense that player one's commitment payoff is equal to his minimax payoff[4] the answer is no as is shown in the following proposition:

> **Proposition 3.1:** Let g be a non trivial game which is not of conflicting interests. Then for any $\epsilon > 0$ there is an $\eta > 0$ and a $\underline{\delta}_2 < 1$ such that the following holds: There is a perturbation of g, in which the commitment type of player one has positive probability and the normal type has probability $(1-\epsilon)$, and there is an equilibrium of this perturbed game, such that the limit of the average payoff of the normal type of player one for $\delta_1 \to 1$ is smaller than $g_1^* - \eta$ for any $\delta_2 > \underline{\delta}_2$.

<u>Proof:</u> See Appendix.

Proposition 3.1 shows that the condition of conflicting interests is not only sufficient but also necessary for Theorem 3.2 to hold. Note that the statement of the proposition is a little bit stronger than necessary. Theorem 3.2 only requires that $\mu(\omega^0) > 0$ in the perturbed game. So we could have established necessity by constructing a perturbation which gives a high prior probability to an "indifferent" type who credibly threatens to punish any deviation of player two from the equilibrium path we want to sustain. This is how we proceeded in the construction of the counterexample of Section 3.3 and it should become clear from the proof of Proposition 3.1 that this would have been possible here as well. However, in many economic applications it is natural to assume that $\mu(\omega^0)$ is close to one. This is why we provide a stronger proposition which says that even if $\mu(\omega^0)$ is arbitrarily close to one it is possible to construct an equilibrium in which the payoff of the normal type of player one is bounded away from his commitment payoff in any game which is not of conflicting interests.

[4] It is well known that a player can always guarantee himself at least his minimax payoff in any Nash-equilibrium.

In the proof of Proposition 3.1 we use (as in our counterexample) the possible existence of a third type who is indifferent between all his strategies no matter what player two does. The reader might ask whether a similar result as Proposition 3.1 can be established using the normal and the commitment type only. The answer is no, that is, there are two long-run player games with no conflicting interests, which are perturbed such that the only additional type is the commitment type, in which player one gets his commitment payoff in all equilibria.[5] However, as discussed in the introduction, we want to insist on "robustness" in the sense that our result should hold no matter what other "crazy" types may be around with positive probability. This is why we don't want to put any restrictions on the set of possible types.

3.4.3. Two-Sided Incomplete Information

If there are two long-run players it is most natural to ask what happens if there is two-sided uncertainty. Our result can be extended to this case as follows. Suppose the game is perturbed in such a way that there is incomplete information about both the payoff functions of player one and player two. Let ω_i denote player i's type which is drawn by nature in the beginning of the game out of the countable set Ω_i according to the probability measure μ_i, $i \in \{1, 2\}$. ω_i^0 and ω_i^* represent the normal and the commitment types, respectively. Finally, suppose that the game is of conflicting interests in the sense that player i's commitment strategy holds player j down to his/her minimax payoff. Now consider the normal type of player two. In the proof of Lemma 3.2 we didn't say why player two chooses an action which is not a best response against player one's commitment strategy. She might do so because she wants to test player one's type or because she wants to build up a reputation

[5] See e.g. Aumann and Sorin (1989). Their automaton with recall zero is equivalent to our commitment type.

herself. No matter what the reason is, Lemma 3.2 states that if she takes $s_2 \notin B(s_1^*)$, then she must expect that player one chooses $s_1 \neq s_1^*$ in one of the following periods with strictly positive probability. This holds for the normal type of player two no matter what other possible types of her exist with positive probability.

A possible strategy of player one still is always to choose s_1^*. If he faces the normal type of player two, then by Theorem 3.2 there are at most $k(\mu^*, \delta_2)$ periods in which player two will not play a best response against s_1^*. In the worst case for player one this happens in the first k periods of the game. On the other hand, if he doesn't face the normal type of player two his expected payoff is at least $\underline{g}_1$ in every period. This gives a lower bound for the expected payoff of the normal type of player one in period 0. Of course, the same argument goes through for player two.

Proposition 3.2: Let g be of conflicting interests and let $\mu_i(\omega_i^0) = \mu_i^0 > 0$ and $\mu_i(\omega_i^*) = \mu_i^* > 0$. Then there are constants $k_i(\mu_i^*, \delta_j)$ otherwise independent of $(\Omega_i, \Omega_j, \mu_i)$, such that

$$V_i(\delta_i, \delta_j, \mu_i^*, \mu_j^0; \omega_i^0) \geq \mu_j^0 \left[\sum_{t=1}^{k_i(\mu_i^*, \delta_j)} \delta_i^{t-1} \underline{g}_i + \sum_{t=k_i(\mu_i^*, \delta_j)+1}^{\infty} \delta_i^{t-1} g_i^* \right] + (1 - \mu_j^0) \underline{g}_i, \tag{3.25}$$

where $V_i(\delta_1, \delta_2, \mu_i^*, \mu_j^0, \omega_i^0)$ is any equilibrium payoff of player one with type ω_0 in any Nash equilibrium of $G^\infty(\mu)$.

If the probability of the normal type of player j is close to 1 and if player i is very patient, then the lower bound for his average payoff is close to his commitment payoff. Of course, in a game with conflicting interests it is impossible that both parties get their commitment payoffs. What matters is the relative patience of the two players. The bigger player j's discount factor the bigger is $k_i(\mu_i^*, \delta_j)$, i.e. the number of periods in which player i must expect that a strategy other than the best response against his commitment strategy is played, and the lower is his lower bound. But if we keep δ_j fixed and take δ_i sufficiently close to 1, then player i will get on average almost his commitment payoff.[6]

3.5. Examples

3.5.1. The Chain Store Game

Consider the classical chain store game, introduced by Selten (1978), with two long-run players. In every period the entrant may choose to enter a market (I) or to stay out (O), while the monopolist has to decide whether to acquiesce (A) or to fight (F). Assume that the payoffs of the unperturbed game are given as follows:

	I	O
A	1, 2	3, 0
F	0, -1	3, 0

Figure 3.2: The chain store game.

The monopolist would like to commit to fight in every period which would give him a commitment payoff of 3 and which would hold the entrant down to 0. Since 0 is also player two's minimax payoff the game is - according to our definition - of conflicting interests. This finitely repeated two long-run player game with some incomplete information about the monopolist's type has been studied by Kreps and Wilson (1982a). For a particular perturbation of player one's payoff function they have shown that there are sequential equilibria in which the monopolist gets

6) See also the following example in Section 5.1.

on average almost his commitment payoff if his discount factor is close enough to one and if there are enough repetitions. However, Fudenberg and Maskin (1986) demonstrated that any feasible payoff vector which gives each player more than his minimax payoff, i.e. any point in the shaded area of figure 2, can be sustained as an equilibrium outcome it the "right" perturbation has been chosen. Thus, our Theorem 3.2 considerably strenghtens the result of Kreps and Wilson (1982a). It says that the only Nash equilibrium outcome of this game which is robust against any perturbation of player one's payoff function gives player one his commitment payoff (note that he cannot get more), and it unequivocally predicts the point $(3, 0)$ to be played on average if player one is sufficiently patient. Furthermore it shows that this result carries over to the infinitely repeated game.

Now suppose that there is some incomplete information about the payoff function of the entrant. She would like to commit to enter in every period which would give her a commitment payoff of 2 while it would hold the monopolist down to 1, his minimax payoff. So again, the game is of conflicting interests and our theorem applies. If there is two-sided uncertainty Proposition 3.2 says that it all depends on the relative patience of the two players and the prior probability distribution. If player one is sufficiently more patient than player two and if the probability of the normal type of player two is close to one, then player one will get his commitment payoff in any Nash equilibrium, and vice versa.

3.5.2. A Repeated Bargaining Game

Suppose there is a buyer (b) and a seller (s) who bargain repeatedly in every period on the sale of a perishable good. The valuation of the buyer is 1 and the production costs of the seller are 0. To make the stage game a simultaneous move game suppose there is a sealed bid double auction in every period: Both players simultaneously submit bids p_b and p_s, $p_i \in \{\frac{1}{n}, \frac{2}{n}, \cdots \frac{n}{n}\}$, and there is trade at price $p = \frac{p_b + p_s}{2}$ if and only if $p_b \geq p_s$. Consider the commitment strategy of the buyer. He would like to commit himself to offer $p_b^* = \frac{1}{n}$ in every period. The unique best reply of the seller is $p_s = \frac{1}{n}$, which gives the seller $g_s^* = \frac{1}{n}$, her minimax payoff. Suppose the payoff function of the buyer is perturbed such that with some positive probability he will always offer p_b^*. Then Theorem 3.2 applies and the buyer will get almost his commitment payoff of $\frac{n-1}{n}$ on average in any Nash equilibrium if his discount factor is close to one.

Note however, that this example is not as clear-cut as the chain store game. We have to assume that there is a minimal bid $\frac{1}{n} > 0$. If the buyer could offer $p_b = 0$ he could hold the seller down to a minimax payoff of 0. But if she gets 0 the seller is indifferent between all possible prices, so she might choose $p_s > 0$ and we end up with no trade. The point is that bargaining over a pie of fixed size is not quite a game of conflicting interests. Some cooperation is needed to ensure that trade takes place at all.

In the last chapter we considered a more complex extensive form game of repeated bargaining with one-sided asymmetric information, which confirms the above result that the informed player can use the incomplete information about his type to credibly threaten to accept only offers which are very favourable to him. There, however, we took a different approach and it is interesting to compare the two models. In Chapter 2 we did not allow for "all possible" but only for "natural" perturbations of player one's payoff function, i.e. we assumed that there may be incomplete information about the seller's costs, $c \in [0, 1]$. We show that the buyer will try to test the seller's type at most a fixed finite number of times and that he will only do so in the end of the game. Surprisingly (from the point of view of Theorem 3.2) we can show that the seller will get his commitment payoff even if he is much <u>less patient</u> than the buyer, so the relative discount factors are not as crucial as here. Furthermore the bargaining game we consider there is not of conflicting interests. There are common interests as well, because players have to cooperate to some extent in order to ensure that trade takes place. The reason why conflicting interests are not a necessary condition is that we did not allow for all possible perturbations.

3.5.3. Games with Common and Conflicting Interests

“Pure” conflicting interests are a polar case and in most economic applications there are both - common and conflicting - interests present. Consider for example a repeated prisoner’s dilemma with the following stage game payoffs:

	C	D
C	2, 2	0, 3
D	3, 0	1, 1

Figure 3.3: The prisoner’s dilemma.

In a formal sense this game is of conflicting interests, but our theorem has no bite. Given that player two takes a best response against his commitment action player one would like most to commit himself to play D(efect) in every period. This holds player two down to her minimax payoff, but it only gives player one his minimax payoff as well. So, trivially he will get at least his commitment payoff in every Nash equilibrium. In this game the problem is not to commit to hold player two down to her minimax payoff, but to commit to cooperate.

Another interesting example is a repeated Cournot game. Here in the stage game firm one would like to commit always to choose the “Stackelberg-leader” quantity, which maximizes his stage game profit given that firm two chooses a best response against it. However, the "Stackelberg-follower" payoff of firm two is positive and thus greater than her minimax payoff, because she can be held down to zero profits if player one gluts the market. So our result does not apply. Again the problem of player one is not to hold player two down as far as possible. Both players have the common interest to maximize joint profits, but interests are also conflicting in the sense that each of them would like to get more for himself at the expense of the other. Here the mere existence of a commitment type doesn't help.

3.6. Extensions and Conclusions

To keep the argument as clear as possible we considered a very simple class of possible stage games with two players only, finite strategy sets, a countable set of possible types, and commitment types who always take the same pure action in the repeated game. All of these assumptions can be relaxed without changing the qualitative results. Fudenberg and Levine (1989) provide a generalization to n-player games in which the strategy sets are compact metric spaces and in which there is a continuum of possible types of player one. In Fudenberg and Levine (1991) they show that the argument can be extended to the case where the commitment types play mixed strategies and to games with moral hazard, in which not the action of player one itself but only a noisy signal can be observed by player two in every period. Since the technical problems involved in the generalizations are the same as in our model we refer to their work for any formal statements and proofs.

Fudenberg and Levine (1989) also demonstrated that the assumption that the stage game is simultaneous-move cannot be relaxed without an important qualification of their Theorem 1. The problem is that in an extensive form game player two may take an action after which player one has no opportunity to show that his strategy is the commitment strategy. Consider for example a repeated bargaining game in which in every period the buyer has to decide first whether to buy or not and then the seller has to choose whether to deliver high or low quality. If the buyer decides not to buy then she will not observe whether the seller would have produced high quality. This is why the seller might fail to get his commitment payoff in equilibrium. Note however that this problem does not arise in our context. The definition of a game with conflicting interests assumes that the commitment strategy of player one holds player two down to her minimax payoff. Therefore, if player two takes an action s_2 in equilibrium after which player one's commitment strategy s_1^* is observationally equivalent to some other strategy $s_1 \neq s_1^*$, then player two cannot get more than her minimax payoff. So s_2 must have been an element of $B(s_1^*)$. However player one's commitment payoff is defined as $g_1^* = max_{s_1 \in S_1} \, min_{\sigma_2 \in B(s_1^*)} \, g_1(s_1, \sigma_2)$. So if player two chooses $s_2 \in B(s_1^*)$ player one cannot get less than g_1^*. Therefore, following Theorem 2 of Fudenberg and Levine (1989) it is straightforward that our result holds without qualification if g is any finite extensive form game.

Finally, the reader might ask what happens if player one would like to commit himself to a more complex strategy which prescribes to take different actions in different periods and which may be conditional on player two's past play. It is easy to check that replacing s_1^* by $s_1^*(h_t)$ doesn't change anything in the proofs of Lemmata 3.1 and 3.2 and Theorem 3.2. However, in a game with conflicting interests very little is gained by this generalization because $s_1^*(h_t)$ has to hold player two down to her minimax payoff in any period and after any history h_t.

To conclude, this chapter has shown that "reputation effects" can explain commitment in a repeated game with two long-run players if and only if the game is of conflicting interests. For this class of games the message of the Folk theorem for games with incomplete information may be misleading. If one of the players is very patient compared to the other player, then any Nash equilibrium outcome which is robust against perturbations of the information structure gives him on average almost his commitment payoff. However, we still know very little about the evolution of commitment and cooperation in games in which both - common and conflicting - interests are present, which clearly is one of the most important issues of future research.

Proof of Lemma 3.2:

Consider any equilibrium (σ_1, σ_2) and fix a history h^t up to any period t along which player one has always played s_1^*, such that h^t has positive probability given (σ_1, σ_2). Such a history exists because $\mu^* > 0$. Suppose that according to the (possibly mixed) equilibrium strategy σ_2^{t+1} player two chooses an action $s_2^{t+1} \notin B(s_1^*)$ in period $t+1$ with positive probability. Suppose further that the probability of player one not playing s_1^* in each of the periods $t+1, t+2, \cdots, t+M$ is smaller than ϵ. It will be shown that this can't be true in equilibrium because then player two would get less than her minimax payoff.

Note that ϵ is independent of t and that M has been chosen in a way to guarantee that $\epsilon > 0$. Define $\pi^\tau(s_1) = Prob(s_1^\tau = s_1 \mid h^{\tau-1})$ and let $V_2^\tau(s_1^\tau, \sigma_2^\tau)$ be the continuation payoff for player two from period τ onwards (and including period τ) given the strategy profile $(s_1^\tau, \sigma_2^\tau)$ in period τ. The expected payoff of player two from period $t+1$ onwards is given by:

$$
\begin{aligned}
V_2^{t+1}(\sigma_1,\sigma_2) \;=\; & \sum_{s_1 \neq s_1^*} \pi^{t+1}(s_1) \cdot V_2^{t+1}(s_1, s_2^{t+1}) + \pi^{t+1}(s_1^*) \cdot \Big\{ g_2(s_1^*, s_2^{t+1}) \\
& + \; \delta_2 \cdot \sum_{s_1 \neq s_1^*} \pi^{t+2}(s_1) \cdot V_2^{t+2}(s_1, \sigma_2^{t+2}) + \delta_2 \cdot \pi^{t+2}(s_1^*) \cdot \Big\{ g_2(s_1^*, \sigma_2^{t+2}) \\
& + \; \cdots + \\
& + \; \delta_2 \cdot \sum_{s_1 \neq s_1^*} \pi^{t+M}(s_1) \cdot V_2^{t+M}(s_1, \sigma_2^{t+M}) + \delta_2 \cdot \pi^{t+M}(s_1^*) \cdot \big\{ g_2(s_1^*, \sigma_2^{t+M}) \\
& + \; \delta_2 \cdot V_2^{t+M+1} \big\} \cdots \Big\} \Big\}.
\end{aligned}
\tag{3.26}
$$

It will be convenient to substract on both sides of the equation $\tilde{g}_2$ in every period. Recall that $\tilde{g}_2$ is the maximal payoff for player two if she takes an action which is not a best response against s_1^*. Then we get:

$$
\begin{aligned}
V_2^{t+1}(\sigma_1,\sigma_2) \ - \ \frac{\tilde{g}_2}{1-\delta_2} \ = \ & \sum_{s_1 \neq s_1^*} \pi^{t+1}(s_1) \cdot \left[V_2^{t+1}(s_1, s_2^{t+1}) - \frac{\tilde{g}_2}{1-\delta_2} \right] \\
& + \ \pi^{t+1}(s_1^*) \cdot \Bigg\{ [g_2(s_1^*, s_2^{t+1}) - \tilde{g}_2] \\
& + \ \delta_2 \cdot \sum_{s_1 \neq s_1^*} \pi^{t+2}(s_1) \cdot \left[V_2^{t+2}(s_1, \sigma_2^{t+2}) - \frac{\tilde{g}_2}{1-\delta_2} \right] \\
& + \ \delta_2 \cdot \pi^{t+2}(s_1^*) \cdot \Big\{ [g_2(s_1^*, \sigma_2^{t+2}) - \tilde{g}_2] \\
& + \cdots + \\
& + \delta_2 \cdot \sum_{s_1 \neq s_1^*} \pi^{t+M}(s_1) \cdot \left[V_2^{t+M}(s_1, \sigma_2^{t+M}) - \frac{\tilde{g}_2}{1-\delta_2} \right] \\
& + \ \delta_2 \cdot \pi^{t+M}(s_1^*) \cdot \big\{ [g_2(s_1^*, \sigma_2^{t+M}) - \tilde{g}_2] \\
& + \delta_2 \cdot \left[V_2^{t+M+1} - \frac{\tilde{g}_2}{1-\delta_2} \right] \big\} \cdots \Big\} \Bigg\}.
\end{aligned}
\tag{3.27}
$$

By assumption the probability that player one does not take his commitment action is smaller than ϵ in any period from $t+1, \cdots, t+M$, so

$$\sum_{s_1 \neq s_1^*} \pi^{t+i}(s_1) \ < \ \epsilon, \tag{3.28}$$

and, of course, we can use that $\pi^{t+i}(s_1^*) \leq 1$. Since $\overline{g}_2$ is the maximal payoff player two can get at all, it has to be true that

$$V_2^{t+i}(s_1, \sigma_2^{t+i}) \ \leq \ \frac{\overline{g}_2}{1-\delta_2} \quad \textit{and} \quad V_2^{t+M+1} \ \leq \ \frac{\overline{g}_2}{1-\delta_2}. \tag{3.29}$$

Furthermore, s_2^{t+1} is supposed not to be a best response against s_1^*, so

$$g_2(s_1^*, s_2^{t+1}) \ \leq \ \tilde{g}_2. \tag{3.30}$$

Finally we can use that $g_2(s_1^*, \sigma_2) \leq g_2^*$. Substituting these expressions yields:

$$
\begin{aligned}
V_2^{t+1}(\sigma_1, \sigma_2) - \frac{\tilde{g}_2}{1-\delta_2} \;&<\; \epsilon \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} \;+\; 1 \cdot \Bigg\{ (\tilde{g}_2 - \tilde{g}_2) \\
&\quad + \; \delta_2 \cdot \epsilon \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} \;+\; \delta_2 \cdot 1 \cdot \Bigg\{ (g_2^* - \tilde{g}_2) \;+\; \cdots \;+ \\
&\quad + \; \delta_2 \cdot \epsilon \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} \;+\; \delta_2 \cdot 1 \cdot \Big\{ (g_2^* - \tilde{g}_2) + \delta_2 \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} \Big\} \cdots \Bigg\} \Bigg\}. \\
&= \; \epsilon \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} + \delta_2 \cdot \epsilon \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} + \delta_2 \cdot (g_2^* - \tilde{g}_2) \\
&\quad + \; \cdots \; + \delta_2^{M-1} \cdot \epsilon \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} + \delta_2^{M-1} \cdot (g_2^* - \tilde{g}_2) + \delta_2^{M} \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} \\
&= \; \epsilon \cdot \big(1 + \delta_2 + \cdots + \delta_2^{M-1}\big) \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} \;+\; \delta_2^{M} \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} \\
&\quad + \; (1 + \delta_2 + \cdots + \delta_2^{M-1}) \cdot (g_2^* - \tilde{g}_2) \;-\; (g_2^* - \tilde{g}_2) \\
&< \; \epsilon \cdot \frac{\overline{g}_2 - \tilde{g}_2}{(1-\delta_2)^2} + \delta_2^{M} \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} - (g_2^* - \tilde{g}_2) + \frac{g_2^* - \tilde{g}_2}{1-\delta_2} \;.
\end{aligned}
\tag{3.31}
$$

Recall from the statement of Lemma 3.2 that

$$
\epsilon \;=\; \frac{(1-\delta_2)^2 \cdot (g_2^* - \tilde{g}_2)}{\overline{g}_2 - \tilde{g}_2} - \delta_2^{M} \cdot (1-\delta_2) \;>\; 0 \;. \tag{3.32}
$$

It is easy to check that ϵ has been chosen such that

$$
\epsilon \cdot \frac{\overline{g}_2 - \tilde{g}_2}{(1-\delta_2)^2} + \delta_2^{M} \cdot \frac{\overline{g}_2 - \tilde{g}_2}{1-\delta_2} \;=\; g_2^* - \tilde{g}_2 \;. \tag{3.33}
$$

Therefore we get:

$$
V_2^{t+1}(\sigma_2) - \frac{\tilde{g}_2}{1-\delta_2} \;<\; \frac{g_2^*}{1-\delta_2} \;-\; \frac{\tilde{g}_2}{1-\delta_2} \;. \tag{3.34}
$$

However, since g_2^* is player two's minimax payoff this is a contradiction to the fact that we are in equilibrium. *Q.E.D.*

Proof of Proposition 3.1:

The proof is similar to the construction of the counterexample in Section 3.3. Perturbe the game g such that there are three types of player one, the normal type, the commitment type and an indifferent type, whose payoff is the same for any strategy profile, with probabilities $(1-\epsilon)$, $\frac{\epsilon}{2}$, and $\frac{\epsilon}{2}$, respectively. Let $\underline{\delta}_2(\epsilon) = \frac{2}{2+\epsilon} < 1$ and suppose $\delta_2 > \underline{\delta}_2(\epsilon)$. Define

$$n \;=\; \frac{ln\left[1 - \frac{2(1-\delta_2)}{\delta_2 \epsilon}\right]}{ln\ \delta_2} \tag{3.35}$$

and let $m = [n] + 2$, where $[n]$ is the integer part of n. Given the restriction on δ_2 it is straightforward to check that n is well defined and positive.

Since the commitment payoff of player one is strictly greater than his minimax payoff there exists a strategy $\tilde{s}_2$ such that $\tilde{g}_1 = g_1(s_1^*, \tilde{s}_2) < g_1^*$ and $\tilde{g}_2 = g_2(s_1^*, \tilde{s}_2) < g_2^*$. Suppose that $\tilde{g}_1 > minmax g_1$ and $\tilde{g}_2 > minmax g_2$.[7] We will now construct an equilibrium such that the limit of the average equilibrium payoff of the normal type of player one for $\delta_1 \to 1$ is bounded away from his commitment payoff by at least η, where

$$\eta \;=\; \frac{1}{m} \cdot [g_1^* - \tilde{g}_1] \;>\; 0. \tag{3.36}$$

Suppose $1 > \delta_1 \geq \sqrt{\frac{\overline{g}_1 - \tilde{g}_1}{\overline{g}_1 - \tilde{g}_1 + g_1^* - \tilde{g}_1}}$, where $\overline{g}_1$ is the maximum payoff player one can get at all. Along the equilibrium path all types of player one play s_1^* in every period, while player two plays $s_2^* \in B(s_1^*)$ in the first $m-1$ periods, then she plays $\tilde{s}_2$ in period n, then starts again playing s_1^* for the next $m-1$ periods and so on. If player one ever deviates from this equilibrium path player two believes that she faces the normal type with probability 1. In this case we are essentially back in a game with complete information where the Folk theorem tells us that any individually rational,

[7] If for any of the players $\tilde{g}_i \leq minmax g_i$ the construction of the "punishment equilibria" which are used below to deter any deviation form the equilibrium path are slightly more complex. In this case players have to alternate between the outcomes g^* and $\tilde{g}$ such that both get on average at least their minimax payoff.

feasible payoff vector can be sustained as a subgame perfect equilibrium. So without writing down the strategies explicitly we can construct a continuation equilibrium, such that the continuation payoff is $\left(\frac{1}{1-\delta_1}\tilde{g}_1, \frac{1}{1-\delta_2}\tilde{g}_2\right)$. Clearly, the commitment and the indifferent type of player one have no incentive to deviate since s_1^* is at least weakly dominant for both of them. It is easy to check that - given $m \geq 2$ and the restriction on δ_1 - the normal type of player one will not deviate either.

Now suppose player two ever deviates in any period t. In this case the normal and the commitment type are supposed to play s_1^* in period $t+1$, while the indifferent type switches to another strategy $\tilde{s}_1^{t+1} \neq s_1^*$. If player two does not observe s_1^* being played in period $t+1$ she puts probability one on the indifferent type. Using the Folk theorem we can construct a continuation equilibrium in this subform which gives player two $\frac{1}{1-\delta_2}\tilde{g}_2$ and which would give the normal type of player one $\frac{1}{1-\delta_1}\tilde{g}_1$. If, however, player two observes s_1^* being played in period $t+1$ she puts probability 0 on the indifferent type. In the continuation equilibrium of this subform (s_1^*, s_2^*) are always played along the equilibrium path. If there is any deviation by player one, player two believes that she faces the normal type with probability one and - using the Folk theorem again - the continuation payoff is $\left(\frac{1}{1-\delta_1}\tilde{g}_1, \frac{1}{1-\delta_2}\tilde{g}_2\right)$. Clearly, always to play s_2^* is a best response of player two against always s_1^* and always s_1^* is a best response for the commitment type against any strategy. It is easy to check that it is also a best response for the normal type of player one, given the "punishment" after any deviation.

We have already shown that the strategies of the players form an equilibrium after any deviation from the equilibrium path and that given the continuation equilibria player one has no incentive to deviate from this path. We still have to check that player two's strategy is a best response along the equilibrium path. The best point in time for a deviation is when player two is supposed to play $\tilde{s}_2$. If it does not pay to deviate in this period, it never will. Suppose player two does not deviate.

Then her payoff is given by:

$$\begin{aligned} V_2(\tilde{s}_2) \;&=\; \tilde{g}_2 \;+\; \sum_{t=1}^{m-1} \delta_2^t g_2^* \;+\; \delta_2^m \tilde{g}_2 \;+\; \sum_{t=m+1}^{2m-1} \delta_2^t g_2^* \;+\; \cdots \\ &=\; \tilde{g}_2 \;+ \frac{\delta_2}{1-\delta_2} \cdot g_2^* \;-\; \frac{\delta_2^m}{1-\delta_2^m} \cdot (g_2^* - \tilde{g}_2)\;. \end{aligned} \tag{3.37}$$

However, if she deviates, the best she can do is to play s_2^* in period t. In this case her payoff is given by

$$V_2(s_2^*) \;=\; g_2^* \;+\; \delta_2 \cdot \left\{ (1-\frac{\epsilon}{2}) \cdot \frac{1}{1-\delta_2} \cdot g_2^* \;+\; \frac{\epsilon}{2} \cdot \frac{1}{1-\delta_2} \cdot \tilde{g}_2 \right\} \tag{3.38}$$

It is now easy to check that ϵ and $\underline{\delta}(\epsilon)$ have been constructed such that $V_2(\tilde{s}_2) > V_2(s_2^*)$. Thus we have established that this is indeed an equilibrium path.

We now have to show that along this equilibrium path the average payoff of the normal type of player one is indeed smaller than $g_1^* - \eta$ when $\delta_1 \to 1$. The equilibrium payoff of the normal type is given by:

$$\begin{aligned} V_1 \;&=\; \sum_{t=1}^{m-1} \delta_1^{t-1} g_1^* \;+\; \delta_1^{m-1} \tilde{g}_1 \;+\; \sum_{t=m+1}^{2m-1} \delta_1^{t-1} g_1^* \;+\; \cdots \\ &=\; \frac{1}{1-\delta_1} \cdot g_1^* \;-\; \frac{1}{\delta_1} \cdot \frac{\delta_1^m}{1-\delta_1^m} \cdot [g_1^* - \tilde{g}_1]\;. \end{aligned} \tag{3.39}$$

Therefore the difference between his commitment payoff and his average payoff in this equilibrium is

$$\begin{aligned} g_1^* \;-\; (1-\delta_1) \cdot V_1 \;&=\; g_1^* - g_1^* + \frac{1-\delta_1}{\delta_1} \cdot \frac{\delta_1^m}{1-\delta_1^m} \cdot [g_1^* - \tilde{g}_1] \\ &=\; \frac{(1-\delta_1) \cdot \delta_1^{m-1}}{1-\delta_1^m} \cdot [g_1^* - \tilde{g}_1] \\ &=\; \frac{(1-\delta_1) \cdot \delta_1^{m-1}}{(1-\delta_1) \cdot \frac{1-\delta_1^m}{1-\delta_1}} \cdot [g_1^* - \tilde{g}_1] \\ &=\; \frac{\delta_1^{m-1}}{\sum_{t=0}^{m-1} \delta_1^t} \cdot [g_1^* - \tilde{g}_1] \;>\; \frac{\delta_1^{m-1}}{m} \cdot [g_1^* - \tilde{g}_1]\;. \end{aligned} \tag{3.40}$$

Taking the limit for $\delta_1 \to 1$ we get

$$\lim_{\delta_1 \to 1} \frac{\delta_1^{m-1}}{m} \cdot [g_1^* - \tilde{g}_1] \;=\; \frac{1}{m} \cdot [g_1^* - \tilde{g}_1] \;=\; \eta\;. \tag{3.41}$$

Q.E.D.

Chapter 4:

The Costs and Benefits of Privatization

4.1. Introduction

Privatization has been one of the major and most controversial economic issues throughout the last decade. However, economic theory still has some difficulties to explain what the difference between a privatized and a nationalized firm is. Usually the analysis falls back on assuming that there are some exogenously given differences in the abilities of the government and the private owner to show that a privatized firm may produce more efficiently than a nationalized one.[1)] While there is certainly a lot of casual empirical evidence to support these assumptions it would be more satisfactory to explain the differences between the two organizational modes endogenously.

The purpose of this chapter is to suggest an alternative approach. Following the modern theory of vertical integration and organizational design we argue that privatization and nationalization can be seen as different "governance structures" (Williamson (1985)). In a world in which only incomplete contracts are feasible the choice of a governance structure matters because it determines who has the residual right to control the firm in all contingencies which have not been dealt with in an explicit contract before. We show that different governance structures give rise to different commitment possibilities of the government and different incentives for the management which can explain some of the costs and benefits of privatization endogenously.

[1)] See Bös (1990) and Vickers and Yarrow (1988) for recent surveys of this literature. Many of the conventional arguments and assumptions are critically summarized in Laffont and Tirole (1990).

The analysis focusses on two efficiency arguments which are commonly used in the literature. Let us briefly explain why the traditional theory has problems to justify them. The first argument deals with "productive efficiency" and claims that production is organized and carried out more efficiently in a privatized than in a public firm because better incentives can be given to managers and workers. This argument has been challenged by Williamson's (1985) idea of selective intervention. Why can't the government reach the same productive efficiency by just mimicking the private owner? If the government organizes the firm in exactly the same way as a private owner would do, if it gives the same incentive schemes to managers and workers, and if it deviates from this policy only if there is a possibility to do something strictly better than the private owner, then a nationalized firm should produce at least as efficiently as a privatized one. The second argument is concerned with "allocative efficiency" and says that a public firm will choose a socially more efficient production level because the government cares about social welfare, whereas a private owner just maximizes his private profits. However, this need not be the case if the government can regulate the firm. Sappington and Stiglitz (1987) suggest a privatization and regulation procedure which perfectly overcomes the problem of different objective functions. The government could auction a contract entitleing the private owner to receive a payment for his output which exactly equals its social valuation. Thus, he completely internalizes social welfare and chooses a socially efficient production level. Furthermore, if the bidding process is competitive, the government will extract all the rents from this contract through the auction even if it does not know the cost function of the firm.

Obviously, any theoretical explanation of the differences between a privatized and a nationalized firm has to explain why the suggestions of Williamson and Sappington and Stiglitz cannot be applied. Note that their arguments are based on a very strong implicit assumption. It has to be possible to write complete contingent contracts for the entire horizon of the firm − otherwise the involved commitment problems cannot be overcome. For example, the government must be able to commit at the stage of privatization to actually pay the social valuation of output to the private owner in the (possibly distant) future. That is, it must be possible to specify unambigiously the social benefit of production for all possible states of the world in a contract such that this agreement can be enforced by the courts. Otherwise the private owner will rationally expect that once he has made a relationship specific investment the government will exploit the fact that investment costs are sunk and expropriate his quasi-rents. So he will not invest efficiently. However, if complete contracts are feasible it is not surprising that there is no difference in efficiency since it is well known that any organizational mode can be copied by any other organizational mode through a complete contingent contract.[2] Therefore, if there is any difference it must be due to the fact that only *incomplete contracts* are feasible at the stage of privatization.

Most of the literature on incomplete contracts follows Grossman and Hart (1986) in assuming that different governance structures (or different allocations of ownership rights) lead to different allocations of bargaining power at later stages of the relationship between the involved parties which in turn affect their investment incentives.[3] However, we will follow a different line of reasoning which is more closely adapted to the privatization context. We argue that the allocation of ownership rights has an important impact on the distribution of inside information about the firm. In particular the government will be better informed about costs and profits if it owns and controls the firm than if it has sold it to a private owner.[4] We show that different information structures give rise to different information rents

[2] See Williamson (1985) and Grossman and Hart (1986).
[3] See Chapter 1.3 for a discussion of the incomplete contracts approach
[4] That incomplete information of the government is a frequent and very serious problem is well known and extensively discussed in the regulation literature. See. e.g. Baron and Myerson (1982) or Caillaud et. al. (1988). and the references given there.

which in turn affect both, investment and production decisions.

The relation between the allocation of ownership rights and the distribution of information has to be explained in some more detail. An obvious fact is that the owner of a firm has privileged access to most information available in his firm (not necessarily to all information) from which he may exclude outsiders. Nevertheless, one might argue that the right to have privileged access to inside information is not necessarily a residual but rather a specific right, i.e. it could be possible to sell it to an outsider. However, if complete contracts are not feasible this will be extremely difficult. The reason is that the information is not just "available" but that it has to be produced in the firm, it has to be collected, accounted, processed, and transmitted, and it is the owner who in the last end controls this process of information production. Therefore the owner is always able to manipulate the information. For example he may manipulate transfer prices, thus shifting profits from one division of his firm to another, or he may choose among different depreciation methods, thus shifting profits between periods, etc..[5] The crucial point is that after the information has been produced it is impossible to verify it to an outsider even if the owner wishes to do so.[6]

How does the distribution of information affect the efficiency of the firm? In our model the information structure can be seen as a commitment device of the gov-

[5] The importance of these problems has been stressed by Williamson (1985, p. 139), Hart (1988, p. 126) and Holmström and Tirole (1989, p. 74f), but they do not elaborate on this theme in a formal model. See also the excellent discussion on "Information structure and comparative organization" in Riordan (1990, p. 105f) where further references are given.

[6] Note however that it might be possible to sell the right of privileged access to inside information ex ante, i.e. before it has been produced. For example the owner could write a contract with an outsider, saying that the outsider is allowed to send his own auditor into the firm who supervises the process of information production. Grossman and Hart (1986, p. 695 and fn3) are referring to this possibility when they say that integration need not change the information structure. However, there is no contradiction to their argument. We only claim that if parties decided not to write an ex ante contract on the distribution of information then they cannot change the distribution ex post.

ernment. The government may privatize because it deliberately wants not to have precise information about the costs and profits of the firm. Very roughly speaking the idea is the following. Under *nationalization* the government is well informed about cost conditions and chooses a socially efficient production level. However, it cannot commit to an incentive scheme which induces the manager to invest efficiently into cost reduction in the set up phase of the project. Thus, while allocative efficiency can be achieved, productive efficiency is poor, because the manager – anticipating that his investments will be expropriated – invests to little. Under *privatization* the government does not know the cost parameters of the firm while the private owner does. It is shown that the optimal subsidy scheme under incomplete information pays an information rent to the private owner if costs are low and distorts production below the socially efficient level if costs are high. Thus, allocative efficiency is clearly worse compared to nationalization but productive efficiency may be enhanced. To show this, we distinguish two different scenarios. In the first one the project is sold to a private "owner-manager" who runs the firm himself. Still, the government cannot commit in an ex ante contract to reward the owner-manager for his cost reducing investments. Nevertheless, he will invest more than under nationalization because this increases his expected information rent. In the second scenario the firm is sold to a "rentier-owner" who (like the government) has to employ a manager to invest and to carry out production. In this case the information rent, paid to the owner, is useless as an incentive device for the manager. However, the manager is induced to invest more, because he suffers from too low production if costs turn out to be high. For example, under privatization the firm will be closed down earlier (i.e. given lower cost realizations) than under nationalization, so the manager faces a "harder budget constraint".[7] Thus, there is a trade off between allocative and productive efficiency which may explain some of

[7] There is an interesting parallel to the "soft budget constraint" argument by Kornai (1979), which is discussed in more detail in Section 4.4.

the costs and benefits of privatization.

The idea that ex post inefficiencies can be desirable because they may improve ex ante efficiency is quite general and well known in the literature. The problem is how to make the ex post inefficiencies renegotiation-proof. Here incomplete information plays a crucial role as has been observed by Dewatripont (1988). There are a few other papers which consider the possibility to choose the information structure of a game through organization design. Dewatripont and Maskin (1989) show in an insurance context with hidden information that it may be desirable ex ante to restrict the set of observable variables in order to improve risk sharing if insurance contracts can be renegotiated. Tirole (1986) was the first to show that information rents can give investment incentives. This idea has been used by Riordan (1990, 1991) to explain the optimal degree of vertical integration in a model closely related to our model of privatization to an owner-manager. The main technical difference is that we allow for more general production and incentive schemes. Finally Laffont and Tirole (1990) and Shapiro and Willig (1990) also link the privatization issue to the problem of asymmetric information. Laffont and Tirole focus on the double marginalization problem after privatization. The government induces the owner to induce the manager to produce, which leads to less effort of the manager than if he were induced directly by the government. Shapiro and Willig deal with a richer institutional framework. In their model a "social planner" has to decide whether to nationalize the firm in which case it is run directly by a minister or whether to privatize it in which case a regulator may influence the production decision of the private owner through taxes and subsidies. Both, the minister and the regulator, may have different private information and private agendas not known ex ante to the social planner. The tradeoff between privatization and nationalization is derived from the differences in objectives and information. Although their model is very different from the one considered here it also stresses the importance of incomplete information.

The rest of this chapter is organized as follows. Section 4.2 describes the model and discusses the contractual and technological assumptions we are going to make. In Section 4.3 the case of privatization to an owner-manager, who runs the firm himself, is dealt with. Section 4.4 considers the more realistic case of privatization to a rentier-owner or an anonymous group of shareholders. Finally, Section 4.5 discusses the possibility of renegotiation of the regulation scheme.

4.2. Description of the Model

Consider a firm which produces a good the quantity of which is y. Production of y yields a "social benefit" $b(y)$ which enters directly the payoff function of the government. The most straightforward interpretation is the provision of a non-marketable public good. However the model can easily be extended to the more general case in which production of y yields a market revenue accruing to the owner of the firm and an additional "net" social benefit, which may be thought of as consumer surplus or as a positive external effect on society.[8] Let $c(y, \theta)$ denote the cost function of the firm, where θ is a parameter representing the state of the world. For expositional clarity we assume that there are only two relevant states of the world, i.e. θ may be either "low" (l) or "high" (h), with $c(y, l) < c(y, h)$ for all $y > 0$.

The government has two possibilities to ensure that the good is going to be produced. It can either control production directly in the nationalized firm or it can privatize the firm and offer a sophisticated subsidy scheme to the private owner in order to make production privately profitable. If it privatizes two cases have

[8] See Schmidt (1990b) for an exposition of this case. The model may also be used to analyze the question of whether to regulate or to nationalize a "natural monopoly".

to be distinguished. The private owner may either be an "owner-manager", who manages the firm himself, or he may be a "rentier", who has to hire a manager to carry out production. The first part of this chapter deals with privatization to an owner-manager which is modelled by the game described below. In Section 4.4 we discuss a slightly modified version of this game which covers the case of privatization to a rentier.

In *period 0* the government (G) has to decide whether to nationalize or to privatize the firm. It is assumed that initially all the property rights on the project are owned by the government, i.e. in case of nationalization it just keeps the firm. However, it cannot carry out production itself but has to employ a manager (M). The hiring process will not be modelled explicitly. We assume that there is a competitive market for identical managers on which a wage contract is offered such that the expected utility of the manager under this contract equals his reservation utility $\underline{U}$, which is normalized to 0. The structure of the wage contract is discussed in more detail at the end of this section. In case of privatization the firm is auctioned on a competitive market to an owner-manager who runs the firm himself. The process of privatization will not be modelled explicitly either. Instead it is assumed that because of competition the auction price z just equals the expected profits of the firm.

At the beginning of *period 1* the manager (the employee-manager in case of nationalization or the owner-manager in case of privatization) has to choose an action e which reduces the expected costs of production and which gives a (non-monetary) disutility $\phi(e)$ to the manager. For example, this action may be the level of effort with which the manager sets up the firm and organizes production or it may be a relationship specific investment in his human capital. Although we will frequently call e the manager's "effort" level, it is important to keep in mind that e is an investment, i.e. something the costs of which have to be incurred today while the benefits will be payed off at some (possibly much) later date. The level of e can only be observed by the manager. e has an impact on the expected costs of production by affecting the probability distribution over the possible states of the world. That is, at the end of period 1 nature draws $\theta = l$ with probability $q(e)$ and $\theta = h$ with probability $1 - q(e)$.

In the beginning of *period 2* the realization of θ is observed by the owner and the manager of the firm. There is no possibility to verify θ to any outsider, because the owner controls the accounting of his firm and might have manipulated his accounts as was discussed in the introduction. Thus, in the *subgame after nationalization* the government is the owner and observes θ. Then it has to decide on the level of production $y^n(\theta)$ and on the level of subsidies $s^n(\theta)$ subject to the constraint that the costs of the firm have to be covered. An equivalent but more convenient way to model the production stage under nationalization is to assume that the manager bears the costs of production and that the government can make him a "take it or leave it" offer, saying that if he produces $y^n(\theta)$ he gets the subsidy $s^n(\theta)$. If the subsidy does not cover his costs the manager may quit and no production takes place. Since we want to concentrate on the investment decision of the manager we leave any additional principal-agent-problems in this short-run production process out of account and assume that at this stage there is no problem to induce the manager to carry out production efficiently. In the *subgame after privatization* it is the private owner who observes θ while the government only knows the ex ante probability function $q(\cdot)$. Suppose the government believes that the owner-manager has taken $e = \hat{e}$ in period 1. Again it can make a "take it or leave it" offer to influence the production decision. Referring to the revelation principle the subsidy offer is modelled as a direct mechanism, $\mathcal{M}(\hat{e}) = \{y^p(\theta', \hat{e}),\ s^p(\theta', \hat{e}),\ \theta' \in \{l, h\}\}$, which may be contingent on a report of the private owner, saying that if he reports the cost parameter to be θ' he has to produce $y^p(\theta', \hat{e})$ and gets a subsidy $s^p(\theta', \hat{e})$.

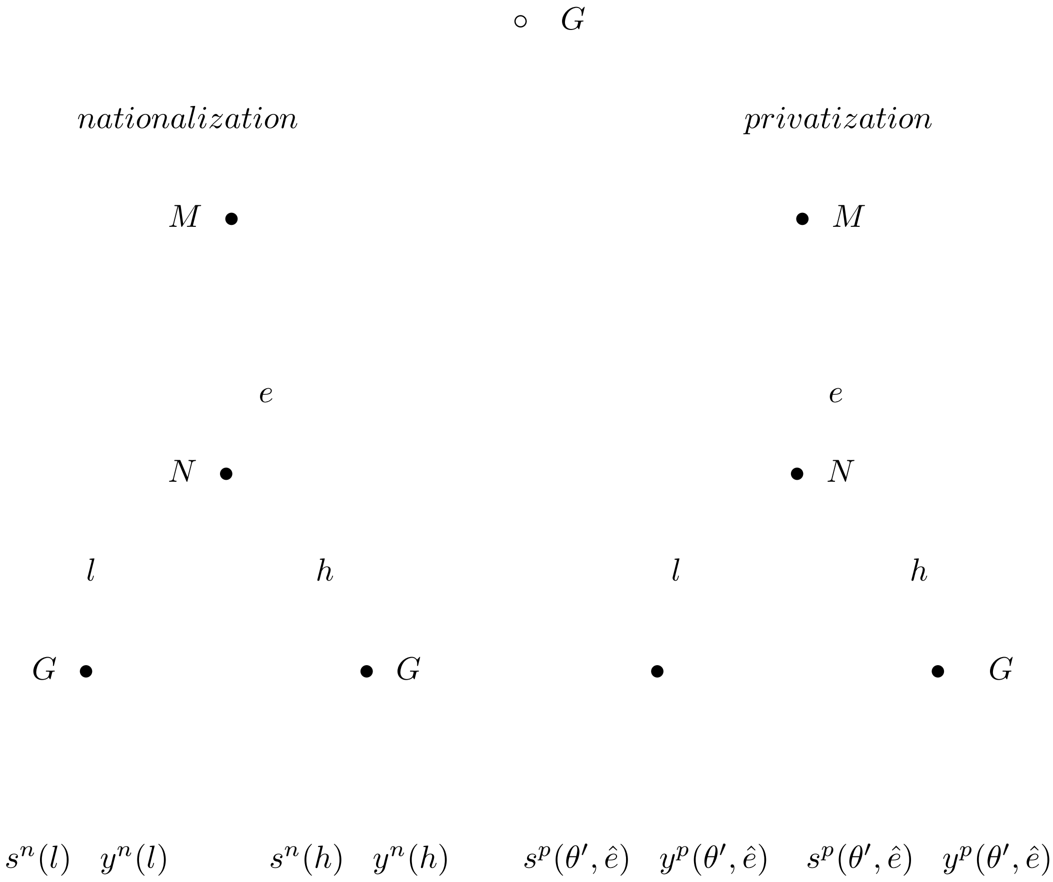

Figure 4.1: Privatization to an owner-manager

However, the government cannot force him to operate under a mechanism which gives him negative profits, so the mechanism has to be accepted voluntarily by the private owner. Such a mechansim is the most general regulation or subsidy scheme one can think of. It is explained in more detail in Section 4.3. At the end of period 2 *payoffs* are realized. Both, the government and the private owner, are assumed to be risk neutral and to maximize their expected utilities:

- The government receives the social benefit of production net of transfers (t) paid to the firm:

$$V \;=\; b(y) - t \;=\; \begin{cases} b(y^n) - w - s^n & \text{after nationalization} \\ b(y^p) + z - s^p & \text{after privatization} \end{cases}$$

If the manager rejects the subsidy scheme in either subgame then $s = y = 0$.[9)]

- The manager's utility function is assumed to be additively separable in his monetary income (m) and his disutility of effort:

$$U \;=\; m - \phi(e) \;=\; \begin{cases} w + s^n - c(y^n, \theta) - \phi(e^n) & \text{after nationalization} \\ -z + s^p - c(y^p, \theta) - \phi(e^p) & \text{after privatization} \end{cases}$$

The structure of the game is summarized in a slightly reduced form in the game tree of *Figure 4.1.* Note that the only difference between the nationalization and the privatization subgame is the different distribution of information, i.e. under public ownership the government observes the cost parameter while under private ownership it does not. In particular, there is no change in the allocation of bargaining power. In both, the nationalization and the privatization subgame the government offers a "take it or leave it" subsidy scheme to the manager. Furthermore, in both subgames the manager is "residual claimant" in the sense that he gets the subsidies and has to pay for the actual costs of production. Without incomplete information under privatization the two subgames would be identical.

If complete contracts could be written in period 0 the first best could easily be achieved in both the nationalization and the privatization subgame. For example, the government could offer a wage contract to the employee-manager such that $w = w(y) = b(y) - \underline{w}$, where $\underline{w}$ is a lump sum transfer from the manager to the government which is chosen such that the expected utility of the manager just equals his reservation utility. Under this wage contract the manager fully internalizes the

9) This very simple specification of the government's payoff function is not crucial. Maximization of total social welfare (including the utility of the manager) doesn't change the qualitative results of our analysis as long as the government is not completely indifferent to transfer payments. The latter assumption is standard in the literature (see e.g. Caillaud et. al. (1986)) and well justified because the government has to raise distortive taxes in order to finance the transfers. Note that the function $b(y)$ is open to different interpretations. It need not be any objective measure of the social benefit of production. It may be whatever gives a positive payoff to the government and what is therefore called "social benefit" even if it reflects some rather narrow interests.

social benefit $b(y)$ in his utility function and chooses the investment level efficiently. Similarly, under privatization the government could write a contract with the owner-manager in period 0 saying that the subsidy to be paid in period 2 is just $s^p(y) = b(y)$ while the auction price z extracts all the expected surplus from the private owner. Again, the owner-manager fully internatilizes the social benefit and chooses e and y efficiently. Of course under these contracts privatization and nationalization are completely equivalent.

Thus, if there is any difference between a nationalized and a privatized firm it has to be due to the fact that complete contracts are not feasible in period 0. We model this, following Grossman and Hart (1986) and the recent literature on ownership and residual control rights, by imposing the following assumption:

> **Assumption 1:** In period 0 no contracts can be written on contingent payments in the future. However, it is possible to choose a governance structure in period 0, i.e. to either nationalize or privatize the firm.

Note that given Assumption 1 only fixed payments w and z are feasible in period 0. Let us justify this contractual assumption in some more detail. Which variables of the model might possibly be controlled by a period 0 contract? Clearly, the effort level e itself cannot be contracted upon because it is only observed by the manager. Furthermore, it is impossible to write a contract conditional on the cost parameter θ or on any accounted costs or profits of the firm. The reason as elaborated in the introduction is that the owner controls the production of this data, so it cannot be verified unambigiously to an outsider (e.g. the courts). Finally it might be possible to circumvent these problems by conditioning the contract on y. However, we have a situation in mind in which both parties do not know in period 0 which type of the good will be the appropriate one to produce in period 2. The relevant type depends on the state of the world which materializes only after the investment decision has been made. This problem is common if the product under consideration needs some further development before it is going to be produced. As an example suppose the government wants to procure a new weapon system ten years from now. Although its precise characteristics will depend on the technological and political development in the next decade, important relationship specific investments have to be made long before the final design becomes clear. In such an environment it would be very costly (if not impossible) to describe precisely the characteristics of the final product in order to contract on y, even so it may easily be described in period 2, i.e. after the state of the world has been realized.[10)]

Finally we impose some technological assumptions which guarantee that the maximization problems of the players have unique solutions which can be characterized by a first-order approach:

Assumption 2: For any $y > 0$, $e > 0$, and $\theta \in \{l, h\}$ it is assumed that $b(y)$, $c(y,\theta)$, $q(e)$ and $\phi(e)$ are twice continuously differentiable, non-negative functions which satisfy

(2.1)	$b(y) \leq \bar{b}$	(2.2)	$b_{yy}(y) < 0$	(2.3)	$b_y(0) > c_y(0,h)$
(2.4)	$c(0,\theta) = 0$	(2.5)	$c_y(y,\theta) > 0$	(2.6)	$c_{yy}(y,\theta) \geq 0$
(2.7)	$c(y,l) < c(y,h)$	(2.8)	$c_y(y,l) < c_y(y,h)$	(2.9)	$c_{yy}(y,l) \leq c_{yy}(y,h)$
(2.10)	$\phi_e(0) = 0$	(2.11)	$\phi_e(e) \geq 0$	(2.12)	$\phi_{ee}(e) > 0$
(2.13)	$0 < q(e) < 1$	(2.14)	$q_e(e) > 0$	(2.15)	$q_{ee}(e) < 0$

Most of these assumptions are self-explanatory. $b(\cdot)$ has been normalized such that $b(0) = 0$. (2.1)-(2.6) ensure an interior solution for the first best production level in all states of the world. (2.7)-(2.9) say that not only costs but also marginal costs are increasing with θ, and that marginal costs increase with y as least as fast in the high

10) The assumption that the level of trade cannot be controlled by a contract in period 0 but can be contracted upon in period 2 is exactly the same as in Grossman and Hart (1986). In footnote 14 they sketch a more formal justification for this assumption along the lines of the argument given here.

cost state than in the low cost state. (2.8) is a "single crossing property", which is standard in the mechanism design literature, while (2.9) is necessary to ensure that the second order conditions are satisfied. Finally, the effort or investment variable e has been normalized such that $e = 0$ is the effort level the manager would spend without any extrinsic incentives. Therefore (2.10) is a quite natural assumption. Together with (2.11)-(2.15) it implies that a positive investment level is desirable given a positive production level in any state of the world.

4.3. Privatization to an Owner-Manager

Before analyzing the game described in the last section let us briefly characterize the first best solution, which could be achieved if complete contracts were feasible, as a point of reference. Expected social welfare is given by

$$W \;=\; q(e) \cdot [b(y(l)) - c(y(l), l)] \;+\; (1 - q(e)) \cdot [b(y(h)) - c(y(h), h)] \;-\; \phi(e). \quad (4.1)$$

For any effort level e and any state of the world θ ex post efficiency requires that

$$y^*(\theta) \;\in\; arg \max_{y(\theta)\geq 0} \{b(y(\theta)) - c(y(\theta), \theta)\}. \quad (4.2)$$

Assumptions (2.1) - (2.6) ensure an interior solution to (2) which must satisfy

$$b_y(y^*(\theta)) = c_y(y^*(\theta), \theta) \quad \theta \in \{l, h\}. \quad (4.3)$$

Note that Assumption (2.8) implies $y^*(l) > y^*(h)$.

The welfare maximizing effort level e^* has to be chosen such that

$$e^* \;\in\; arg \max_{e\geq 0} \{W(e) \;=\; q(e) \cdot W^*(l) \;+\; (1 - q(e)) \cdot W^*(h) \;-\; \phi(e)\}, \quad (4.4)$$

where $W^*(\theta) = b(y^*(\theta)) - c(y^*(\theta), \theta)$. Given Assumptions (2.10)-(2.15) this problem has a unique interior solution characterized by

$$q_e(e^*) \cdot [W^*(l) - W^*(h)] \;=\; \phi_e(e^*), \quad (4.5)$$

i.e. the optimal effort level is chosen such that the marginal costs of effort equal the expected marginal welfare gain given the ex post efficient production scheme $y^*(\theta)$. Thus, we get for the first best level of social welfare

$$W^* = q(e^*) \cdot [b(y^*(l)) - c(y^*(l), l)] \; + \; (1 - q(e^*)) \cdot [b(y^*(h)) - c(y^*(h), h)] \; - \; \phi(e^*). \tag{4.6}$$

4.3.1. Subgame after Nationalization

In the nationalization subgame the government observes θ in the beginning of period 2 before it has to decide on the level of production and the subsidies it is going to offer to the manager. The solution to the bargaining problem in period 2 is straightforward. The manager will accept an offer $\{y(\theta), s(\theta)\}$ if and only if

$$s(\theta) - c(y(\theta), \theta) \; \geq 0, \tag{4.7}$$

where 0 is his normalized outside option utility. Since this constraint will always be binding in equilibrium the government's problem reduces to

$$\underset{y(\theta)}{max} \left\{ V^n = b(y(\theta) - c(y(\theta), \theta) \right\}, \tag{4.8}$$

i.e. to maximize social benefits net of production costs. This is of course the same problem as maximising welfare in the first best case. So the government will choose

$$y^n(\theta) \; = \; y^*(\theta), \tag{4.9}$$

where $y^*(\theta)$ is the ex post efficient production scheme given in (4.3).

Anticipating the production decision of the government the manager in period 1 has to decide on his effort level. Under nationalization, however, the manager is an employee whose wage contract could not be made contingent on his performance or on the realization of costs. Given the fixed wage w the manager maximizes

$U^n = w - \phi(e)$, which is independent of the government's production decision. Not surprisingly it is a dominant strategy to spend no effort additional to what he would do anyway, i.e.

$$e^n = 0. \tag{4.10}$$

Anticipating this the government will offer $w = \underline{U} + \phi(0) = \phi(0)$ in period 0. Therefore, the expected payoff of the government in the nationalization subgame is given by

$$V^n = q(0) \cdot [b(y^*(l)) - c(y^*(l), l)] + (1 - q(0)) \cdot [b(y^*(h)) - c(y^*(h), h)] - \phi(0). \tag{4.11}$$

Although the production level is chosen ex post efficiently this clearly falls short of the first best because the manager does not invest sufficiently into cost reduction and productive efficiency is poor.

4.3.2. Subgame after Privatization

Under privatization the government cannot observe the realization of the cost parameter in the beginning of period 2 but it knows the probability function $q(\cdot)$. Suppose it believes that the manager has taken effort $\hat{e}$ with probability 1. Then the problem of the government is to choose a regulation scheme which maximizes its payoff by inducing the owner-manager to choose a socially more efficient production level. In general such a regulation scheme may be quite complex. However, by the revelation principle (see e.g. Myerson (1979)) it is well known that for any equilibrium of any mechanism there exists a *direct mechanism*, in which players are simply asked to announce their private information and in which truthtelling is an equilibrium strategy, such that the direct mechanism induces the same action of the private owner and gives the same expected payoff to the government. Thus, without loss of generality, the best the government can do is to implement a direct mechanism $\mathcal{M}(\hat{e}) = \{s^p(\theta', \hat{e}), y^p(\theta', \hat{e}), \theta' \in \{l, h\}\}$ based on the report θ' of the private owner about his cost parameter. So the government's problem under privatization is to solve the following program:

$$\max_{y^p(\theta,\hat{e}),s^p(\theta,\hat{e})} \{q(\hat{e}) \cdot [b(y^p(l,\hat{e})) - s^p(l,\hat{e})] + (1-q(\hat{e})) \cdot [b(y^p(h,\hat{e})) - s^p(h,\hat{e})]\} \tag{4.12}$$

subject to

$$s^p(\theta,\hat{e}) - c(y^p(\theta,\hat{e}),\theta) \ \geq s^p(\theta',\hat{e}) - c(y^p(\theta',\hat{e}),\theta) \quad \forall \theta, \theta' \in \{l,h\} \tag{4.13}$$

$$s^p(\theta,\hat{e}) - c(y^p(\theta,\hat{e}),\theta) \ \geq 0 \quad \forall \theta \in \{l,h\} \tag{4.14}$$

Note, that z is a sunk cost for the owner-manager and irrelevant for any decisions in period 2. The incentive compatibility constraint (4.13) ensures that truthtelling is a weakly domininant strategy for all possible cost parameters. The participation constraint (4.14) has to be satisfied because the government cannot force the private owner to take part in the mechanism if he would make negative profits.

This mechanism design problem is by now well understood.[11)] Note first that there is a feasible mechanism which implements the ex post efficient production level, namely $y^p(\theta,\hat{e}) = y^*(\theta)$ and $s^p(\theta,\hat{e}) = b(y^*(\theta))$. Under this subsidy scheme the private owner gets the entire social benefit of production as a monetary reward and internalizes the positive external effect. However, this mechanism is only optimal if the government is indifferent to transfer payments to the private owner. If his rents do not count in the objective function of the government or, more generally, if transfers are costly, this mechanism will not be optimal. In this case the government would prefer most to implement the ex post efficient $y^*(\theta)$ by offering a subsidy which just pays for the uncovered costs and leaves the owner-manager with his reservation payoff. However, such a subsidy scheme would not be incentive compatible because

[11)] See e.g. Baron and Myerson (1982), Besanko and Sappington (1987) or Caillaud et. al. (1988). Note that all of the properties of this very simple mechanism design problem we are going to use in the following analysis carry over to the more general case in which θ is drawn from an interval $[\underline{\theta}, \overline{\theta}]$.

the private owner would always overstate his costs. To induce truthtelling the government faces a tradeoff. It can either pay a higher subsidy if a low value of θ is announced to make it more attractive to report $\theta = l$ truthfully. Or it can distort $y^p(\theta)$ for high values of θ, thus making it less attractive to overstate costs. The optimal mechanism trades off ex post efficiency and costly subsidies.

Proposition 4.1: If the government believes that the manager has taken the effort level $\hat{e}$ with probability 1, then the optimal direct mechanism under privatization is given by:

$$\begin{aligned} s^p(l,\hat{e}) &= c(y^p(l,\hat{e}),l) + c(y^p(h,\hat{e}),h) - c(y^p(h,\hat{e}),l) \\ s^p(h,\hat{e}) &= c(y^p(h,\hat{e}),h) \\ y^p(l,\hat{e}) &= y^*(l) \\ y^p(h,\hat{e}) &= \begin{cases} 0 & \text{if } b(\tilde{y}^p(h,\hat{e})) - c(\tilde{y}^p(h,\hat{e}),h) - \\ & \qquad \frac{q(\hat{e})}{1-q(\hat{e})} \cdot [c(\tilde{y}^p(h,\hat{e}),h) - c(\tilde{y}^p(h,\hat{e}),l)] < 0 \\ \tilde{y}^p(h,\hat{e}) & \text{else} \end{cases} \end{aligned}$$

where $\tilde{y}^p(h,\hat{e})$ is defined by

$$b_y(\tilde{y}^p(h,\hat{e})) = c_y(\tilde{y}^p(h,\hat{e}),h) + \frac{q(\hat{e})}{1-q(\hat{e})} \cdot [c_y(\tilde{y}^p(h,\hat{e}),h) - c_y(\tilde{y}^p(h,\hat{e}),l)]$$

and is strictly decreasing in $\hat{e}$.

Proof: See appendix.

Comparing the optimal production schemes under nationalization and under privatization four observations can be made:

1) $y^p(\theta,\hat{e}) = y^*(\theta)$ if and only if $\theta = l$. This is the "no distortion at the top" property which is well known from the optimal taxation literature.

2) $y^p(\theta,\hat{e}) < y^*(\theta)$ if $\theta = h$ because $q(\hat{e}) > 0$. Thus, production will be inefficiently low under privatization in the high cost state. Although $y^p(h,\hat{e})$ is decreasing in $\hat{e}$ (strictly decreasing if $y^p(h,\hat{e}) > 0$), it does not depend on e, the actual

effort level the manager has taken in period 1. The belief of the government $\hat{e}$ is independent of e because the actual effort level is not observable. But, of course, in equilibrium $\hat{e}$ and e must be equal.

3) It may be optimal to close down the firm under privatization if costs turn out to be high, although this is ex post inefficient. Note that the distortion of production in the high cost state is unavoidable given the incomplete information of the government.[12)]

4) In the high cost state the private owner is held down to his outside option utility, but in the low cost state he makes a positive profit:

$$\pi^p(l,\hat{e}) \;=\; s^p(l,\hat{e}) - c(y^p(l,\hat{e}),l) \;=\; c(y^p(h,\hat{e}),h) - c(y^p(h,\hat{e}),l) \;\geq\; 0. \quad (4.15)$$

Note that $\pi^p(l,\hat{e})$ is increasing with $y^p(h,\hat{e})$, i.e. the smaller the distortion of production the bigger is the profit of the owner-manager. The only economic purpose of $\pi^p(h,\hat{e})$ in period 2 is to induce truthful revelation of $\theta = l$, so it is called the "information rent" of the private owner.

Anticipating the mechanism described above for period 2 the owner-manager has to decide on his effort level in period 1. While there was no incentive to choose a positive e under nationalization this is no longer true in the privatization case. Because the effort level is not observable by the government the manager will take the expectation of the government, $\hat{e}$, as given. Therefore his optimization problem under privatization is:

$$\max_{e \geq 0} \{q(e) \cdot [c(y^p(h,\hat{e}),h) - c(y^p(h,\hat{e}),l)] + (1-q(e)) \cdot 0 - \phi(e)\} \quad (4.16)$$

Suppose that $y^p(h,\hat{e}) > 0$, so that the information rent is strictly positive.[13)] Then we can state the following proposition:

[12)] It could be argued that after the private owner has announced θ truthfully, there is symmetric information and $y^p(\theta)$ could be renegotiated. However, if the private owner anticipates the renegotiation he is no longer induced to tell the truth. Section 4.5 deals with this problem.

[13)] If $y^p(h,\hat{e}) = 0$, the owner-manager gets no information rent and chooses $e^p = 0$.

Proposition 4.2: Under privatization the owner-manager chooses an effort level e^p, $0 < e^p < e^*$, which is characterized by the following first order condition:

$$q_e(e^p)\,[c(y^p(h,\hat{e}),h) - c(y^p(h,\hat{e}),l)] \;=\; \phi_e(e^p). \tag{4.17}$$

There exists a unique e^p satisfying (4.17) with $\hat{e} = e^p$.

Proof: Assumptions (2.10)-(2.15) immediately imply that (16) has a unique interior solution which can be characterized by (17). Given $y^p(h,\hat{e}) > 0$ the information rent of the manager is positive and $e^p > 0$. Furthermore, $e^p < e^*$ iff

$$c(y^p(h,\hat{e}),h) - c(y^p(h,\hat{e}),l) < W^*(l) - W^*(h)\;. \tag{4.18}$$

However, this strict inequality always holds because

$$\begin{aligned} c(y^p(h,\hat{e}),h) - c(y^p(h,\hat{e}),l) \;&\le\; c(y^*(h),h) - c(y^*(h),l) \\ &=\; [b(y^*(h)) - c(y^*(h),l)] \;-\; [b(y^*(h)) - c(y^*(h),h)] \\ &<\; [b(y^*(l)) - c(y^*(l),l)] \;-\; [b(y^*(h)) - c(y^*(h),h)] \\ &=\; W^*(l) - W^*(h) \;\; \Big(< W^p(l) - W^p(h)\Big). \end{aligned} \tag{4.19}$$

Now we prove the last statement of Proposition 4.2. From the proof of Proposition 4.1 it is known that $y^p(h,\hat{e})$ is continious, differentiable and decreasing in $\hat{e}$. Using the implicit function theorem there exists a unique differentiable solution $e^p(\hat{e})$ where $e^p(\hat{e})$ is decreasing in $\hat{e}$, because

$$\frac{de^p(\hat{e})}{d\hat{e}} \;=\; -\,\frac{q_e(e^p)\cdot y^p_{\hat{e}}(h,\hat{e})\cdot[c_y(y^p(h,\hat{e}),h) - c_y(y^p(h,\hat{e}),l)]}{q_{ee}(e^p)\cdot[c(y^p(h,\hat{e}),h) - c(y^p(h,\hat{e}),l)] \;-\; \phi_{ee}(e^p)} \;\le\; 0. \tag{4.20}$$

However, this is due to the simplifying assumption that there are only two states of the world. In the more general case with a continuum of possible states of the world the firm will be closed down earlier under privatization (i.e. for smaller values of θ than under nationalization), but the information rent is always positive as long as there are some states of the world with positive production. See Besanko and Sappington (1987) and Schmidt (1990b).

Furthermore, we know that $e^p(0) \geq 0$ and $e^p(e^*) < e^*$. Therefore, the mean value theorem together with the monotonicity of $e^p(\hat{e})$ imply that there exists a unique e^p satisfying (4.17) at $\hat{e} = e^p$. Q.E.D.

Riordan (1990) considers a similar model on vertical integration in which $y \in \{0, 1\}$. In this special case the manager still underinvests compared to the first best. However, given the expected level of production, his investment is efficient. Note, that this is not true in the more general case with $y \in \mathbb{R}_0^+$ as can be seen from the last (bracketed) inequality in (4.19).

Propositions 4.1 and 4.2 characterize the unique subgame perfect equilibrium of the privatization subgame. Let us now turn to period 0. Given the optimal mechanism of Proposition 4.1 and the optimal choice $e^p(= \hat{e})$ as characterized in Proposition 4.2, the expected payoff of the owner-manager is given by

$$U^p \;=\; q(e^p)\,[c(y^p(h, e^p), h) - c(y^p(h, e^p), l)] \;-\; \phi(e^p). \tag{4.21}$$

Competition in the auction for the firm drives his expected payoff down to his outside option utility, so

$$z \;=\; q(e^p)\,[c(y^p(h, e^p), h) - c(y^p(h, e^p), l)] \;-\; \phi(e^p). \tag{4.22}$$

Substituting $s^p(\cdot, \hat{e})$ by the expressions of Proposition 4.1 and z by (4.22) the expected payoff of the government in period 0 if it decides to privatize is given by

$$V^p \;=\; q(e^p)\,[b(y^*(l)) - c(y^*(l), l)] + (1 - q(e^p))\,[b(y^p(h, e^p)) - c(y^p(h, e^p), h)] - \phi(e^p). \tag{4.23}$$

Note that the government extracts all expected rents from the private owner through the auction. Now we can summarize our main result:

Theorem 4.1: Privatization to an owner-manager is prefered by the government to nationalization if and only if the welfare gain through the more efficient effort decision of the owner-manager outweights the welfare loss due to the ex post inefficient low production under privatization, i.e. iff

$$q(e^p) \cdot [b(y^*(l)) - c(y^*(l), l) + (1 - q(e^p)) \cdot [b(y^p(h, e^p)) - c(y^p(h, e^p), h)] - \phi(e^p)$$
$$\geq \; q(0) \cdot [b(y^*(l)) - c(y^*(l), l)] + (1 - q(0)) \cdot [b(y^*(h)) - c(y^*(h), h)] - \phi(0). \tag{4.24}$$

Comparing the expected payoffs of the government under privatization and under nationalization the costs and benefits of privatization are clear. On the one hand production is carried out more efficiently (higher *productive efficiency*) under privatization because the information rent induces the manager to work harder. From the point of view of the government in period 2 this rent is a pure deadweight loss. However, ex ante it is beneficial because it gives better incentives to the manager. The government cannot offer the same incentives under nationalization because it cannot commit not to expropriate the returns on the manager's investment once it is sunk. It is the asymmetric information in the privatization subgame which is crucial to make the incentives credible. On the other hand, the asymmetric information under privatization causes a distortion of the production level (lower *allocative efficiency*). Note that ex ante the government would like to commit not to distort production in period 2. First of all this would enhance allocative efficiency. Secondly, the information rent (which can be recovered through the auction) increases with $y^p(h)$ and so does the level of effort. However, this commitment is impossible if complete contracts are not feasible.

4.4. Privatization to a Rentier

The analysis of the last section is based on the assumption that the firm is sold directly to the manager. The question arises whether a similar result holds in the more realistic case in which the firm is privatized to a rentier or to a crowd of shareholders who are not directly involved in any productive activity. In this section it is shown that a similar result holds although the argument which drives the model is quite different.

We stick to the assumption that insiders (i.e. manager and owner) are informed about the realization of the cost parameter, while outsiders are not. If the firm is sold to many small shareholders the idea is that they are represented by a board of directors, acting on their behalf, who monitors the management and costlessly observes θ (as the government does under nationalization). In the privatization regime the government now has to deal with the rentier-owner of the firm. It cannot circumvent him and "bribe" the manager in order to get information on θ.[14] So the only change in the game form of Figure 4.1 is that the government now offers the direct mechanism in period 2 to a third player, the rentier-owner, who bears the costs of production and maximizes his profit. Note that without any further modification the privatized firm can only do worse than the nationalized one. The government will still distort production under privatization, but the information rent now goes to the rentier and doesn't induce any additional investment of the manager.

However, there is a countervailing effect if the payoff function of the manager is slightly extended. Suppose that his utility does not only depend on his wage payment and his effort but that it is additionally increasing in the level of production. There are two reasons for this impact. First, the manager incurs a substantial

[14] If this were possible, it would lead us back to the model of Section 4.3.

disutility if the firm is shut down, e.g. because he gets unemployed and has to look for a new job, his reputation may be damaged or he may lose other quasi-rents from his current employment.[15] Second, it is a common claim that managers often behave as "empire-builders" who seek to maximize their budgets, the number of subordinates, power or just their consumption on the job.[16] For both reasons the manager is interested in a higher production level rather than a lower one. Let the utility function of the manager be additively separable in its arguments, i.e.

$$U(w, e, y) \;=\; w - \phi(e) + \psi(y) \tag{4.25}$$

where $\psi(y)$ represents his utility from the production level with $\psi_y(y) > 0$ and $\psi_{yy}(y) < 0$ for all $y > 0$. $\psi(y)$ may be discontinuous at $y = 0$, i.e. if the firm is closed down.[17] Again, only fixed wage contracts are feasible.

4.4.1. Subgame after Nationalization

In the nationalization subgame little changes compared to Section 4.3. Again the government, having observed θ, will choose the production scheme $y^n(\theta) = y^*(\theta)$ given in (4). Because it does not take into account the impact of y on the manager's utility $y^*(\theta)$ is not fully ex post efficient. However, although the impact of $\psi(y)$ may be quite considerable from the point of view of the manager, we assume for simplicity that from a social point of view $\psi(y)$ is negligible, i.e. it does not affect the socially optimal level of production.

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15) The threat of bankruptcy is widely considered to be an important incentive device for the management. It could be endogenized explicitly by adding an efficiency wage model for the managerial labour market in our model.

16) Marris (1963) and Jensen (1986) stress the importance of these management objectives if ownership and control are seperated. See Donaldson (1984) for empirical support of the hypothesis.

17) This more complex utility function of the manager has not been used in Section 4.3 in order to keep the presentation of the argument as clean as possible. However, it would not change any of the qualitative results.

The manager maximizes his utility by choosing the effort level e^n in period 1 characterized by

$$q_e(e^n) \cdot [\psi(y^n(l)) - \psi(y^n(h))] \ = \ \phi_e(e^n). \tag{4.26}$$

Clearly $e^n < e^*$. However, in contrast to Section 4.3, $e^n > 0$. The manager now cares about the expected level of production which can be increased by investing more into cost reduction. Competition on the market for managers drives his wage down to

$$w^n \ = \ \phi(e^n) \ - \ q(e^n) \cdot \psi(y^n(l)) \ - \ (1 - q(e^n)) \cdot \psi(y^n(h)). \tag{4.27}$$

So the expected payoff of the government is given by

$$V^n = q(e^n) \cdot [b(y^*(l)) - c(y^*(l), l)] + (1 - q(e^n)) \cdot [b(y^*(h)) - c(y^*(h), h)] - w^n. \tag{4.28}$$

Note that the government could improve its payoff if it could commit in period 0 to an ex post inefficient subsidy scheme or, equivalently, to an ex post inefficient production scheme. Suppose for example the government could credibly threaten to close down the firm if costs turn out to be high, although this is ex post inefficient. Then the manager might have a strong incentive to work harder in order to reduce the probability that this will actually happen. If the efficiency gain through the more efficient action of the manager is bigger than the efficiency loss due to the distortion of production in the bad state of the world, this commitment would be beneficial.

However, such a commitment is not feasible under nationalization. Once the manager's effort decision is bygone the government will tend to "forgive" high ex post costs and to subsidize the firm in order to achieve the ex post efficient production level. Anticipating this the manager faces a "soft budget constraint" which gives poor ex ante incentives to keep costs low.

4.4.2. Subgame after Privatization

In period 2 the government, not knowing the realization of θ, now offers the direct mechanism to the rentier-owner of the firm. However, the mechanism design problem is exactly the same as in Section 4.3 and the optimal direct mechanism is given by Proposition 4.1. The difference is that the information rent in the good state of the world is now going to the rentier owner and not to the manager who has to decide on the level of cost reducing investment. So this rent cannot be used to promote effort incentives. However, the optimal mechanism also distortes production below its ex post efficient level in the bad state of the world. It is this property which induces the employee-manager to work harder than under nationalization:

Proposition 4.3: Under privatization the employee-manager chooses an effort level e^p, $e^n < e^p < e^*$, which is characterized by

$$q_e(e^p) \cdot [\psi(y^p(l, \hat{e})) - \psi(y^p(h, \hat{e}))] \;=\; \phi_e(e^p) \; . \tag{4.29}$$

There exists a unique e^p satisfying (4.29) with $\hat{e} = e^p$.

<u>Proof:</u> (2.10)-(2.15) and $y^p(l, \hat{e}) - y^p(h, \hat{e}) > 0$ imply that the manager's maximization problem has a unique interior solution characterized by (4.28). Comparing (4.29) and (4.26) and using the characterization of $y^p(\cdot)$ given in Proposition 4.1, it follows that $e^n < e^p$. Since the manager does not take into account the positive external effect of his effort level on social welfare, clearly $e^p < e^*$. The proof of the second part of the proposition is analogous to the second half of the proof of Proposition 4.2. Q.E.D.

Turning now to period 0 the manager will be offered

$$w^p \;=\; \phi(e^p) \;-\; q(e^p) \cdot \psi(y^p(l, e^p)) \;-\; (1 - q(e^p)) \cdot \psi(y^p(h, e^p)) \tag{4.30}$$

and the auction price for the rentier-owner is given by

$$z \;=\; q(e^p) \cdot [c(y^p(h, e^p), h) - c(y^p(h, e^p), l)] \;-\; w^p. \tag{4.31}$$

Substituting these expressions in the payoff function of the government yields

$$V^p \;=\; q(e^p)\,[b(y^*(l)) - c(y^*(l),l)] + (1-q(e^p))\,[b(y^p(h,e^p)) - c(y^p(h,e^p),h)] \;-\; w^p, \tag{4.32}$$

so we can state:

Theorem 4.2: Privatization to a rentier-owner is prefered by the government to nationalization if and only if the welfare gain through the more efficient effort decision of the employee-manager outweights the welfare loss due to the ex post inefficient low production under privatization, i.e.

$$\begin{aligned} q(e^p)\cdot[b(y^*(l)) - c(y^*(l),l)] + (1-q(e^p))\cdot[b(y^p(h,e^p)) - c(y^p(h,e^p),h)] - w^p \\ \geq\; q(e^n)\cdot[b(y^*(l)) - c(y^*(l),l)] + (1-q(e^n))\cdot[b(y^*(h)) - c(y^*(h),h)] - w^n, \end{aligned} \tag{4.33}$$

where

$$w^i \;=\; \phi(e^i) - q(e^i)\cdot\psi(y^i(l)) - (1-q(e^i))\cdot\psi(y^i(h)), \quad i \in \{n,p\}\;. \tag{4.34}$$

In Schmidt (1990b) we give a simple example to show under which parameter constellations privatization is superior to nationalization and vice versa.

It is interesting to compare the driving forces of the models in Section 4.3 and 4.4. The problem of the government is that it cannot commit ex ante to an incentive scheme which induces the manager to invest efficiently into cost reduction. In both models privatization serves as a commitment device for the government, which is credible because of the incomplete information in the privatization regime. There is essentially the same trade off between allocative and productive efficiency. However, the incentives provided through privatization are very different. The *owner-manager* of Section 4.3 is induced to work harder because he expects the information rent in period 2. Through privatization the government deliberately chooses not to be informed about the costs of the firm since this is the only possiblity to commit to actually pay the rent in period 2 - and not to fully expropriate the manager. In contrast to this the *employee-manager* of Section 4.4 is induced to work harder because he is "punished" by the inefficient low production level if costs turn out to be high.[18] Here the government chooses not to be informed about the cost parameter because this is the only credible way to commit to carry out the punishment (i.e. to distort production) although this is ex post inefficient.

The model of this section can be seen as a formalization of the "soft budget constraint" effect observed by Kornai (1979, p. 806f) in centralized economies:

> *"The budget constraint is soft if the state helps the firm out of trouble. There are various means to do so: subsidies; individual exemption from the payment of taxes or other charges ⋯; prolongation of the due credit payment, etc. The state is a universal insurance company which compensates the damaged sooner or later for every loss. The paternalistic state guarantees automatically the survival of the firm.*
>
> *⋯ the hardness or softness of the budget constraint reflects an attitude. ⋯(It) is an ex ante behaviorial regularity, which exerts an influence on the firm's decision."*

It has been shown why the government rationally "forgives" high costs in the public firm, and that privatization may be one way to harden the manager's budget constraint and to change his "attitude". This raises the question whether privatization is the only way to do so. Why can't the government deliberately choose not to be informed about the costs of the public firm by delegating supervision to another third party, e.g. a regulatory agency, which is not an owner. The problem is, that

[18] I am grateful to a referee who made me aware of the following observation. In case of an <u>un</u>regulated monopoly output variablity (in reaction to different cost realizations) is typically lower than in the first best. Hence, the above model implies that productive efficiency in a nationalized firm should be higher than in an unregulated monopoly, while it is lower than in a regulated one.

the government would get the information in period 2 too cheaply. The information rent to be payed to a regulator will be much smaller than the one to a residual claimant on profits. But only if this rent is big enough the government will try to limit it by distorting production.

4.5. Renegotiation of the Optimal Mechanism

The basic problem of the government under nationalization is that it cannot credibly commit to an ex post inefficiency in order to give better ex ante incentives to the manager. However, modelling the privatized firm, we implicitly assumed that the government can commit itself to do something which is ex post inefficient. In period 2 the private owner had to announce his cost parameter θ before he gets $s^p(\theta)$ and produces $y^p(\theta)$. But after he has announced θ truthfully there is symmetric information and no reason why to distort production any more. Thus, $y^p(\theta)$ could be renegotiated to $y^*(\theta)$ such that both, the government and the private owner, are better off. However, if the renegotiation is anticipated by the private owner, it is no longer optimal for him to announce θ truthfully.

Suppose the government does not offer a direct mechanism but a subsidy scheme $s(y)$ to the private owner, saying that if he produces y he gets the subsidy $s(y)$. Furthermore, suppose that the technology is such that after the firm has produced y the production of an additional amount Δy is prohibitively costly, e.g. because of high set-up costs which would have to be incurred again. In this case the commitment not to renegotiate $s(y)$ is provided through the production technology. Proposition 4.4 shows that a subsidy scheme $s(y)$ exists which has the same properties as the optimal direct mechanism given in Proposition 4.1:

Proposition 4.4: There exists a subsidy scheme $s(y, \hat{e})$ such that

(i) $s(y, \hat{e})$ is voluntarily accepted by the private owner,

(ii) $s(y, \hat{e})$ induces the private owner to produce $y^p(\theta, \hat{e})$ in state θ, and

(iii) the payoff of the government given $s(y, \hat{e})$ is the same as the payoff under the optimal mechanism.

<u>Proof:</u> The proof is by construction. Define

$$s(y, \hat{e}) \;=\; \begin{cases} s^p(l, \hat{e}) & \text{if } y = y^p(l, \hat{e}) \\ s^p(h, \hat{e}) & \text{if } y = y^p(h, \hat{e}) \\ -\epsilon & \text{else} \end{cases} \tag{4.35}$$

If the private owner accepts $s(y, \hat{e})$ and produces $y^p(\theta, \hat{e})$ in state θ, then his payoff in each state of the world is nonnegative because the optimal mechanism satisfies the participation constraint (4.14), so he will accept $s(y, \hat{e})$ voluntarily. Given this subsidy scheme and the actual state θ the private owner has to decide on the production level. Clearly any $y \neq y(\theta, \hat{e})$, $\theta \in \{l, h\}$, cannot be optimal because it gives a negative payoff. Furthermore, the incentive compatibility constraint (13) implies that

$$s(y^p(\theta, \hat{e})) - c(y^p(\theta, \hat{e}), \theta) \;\geq\; s(y^p(\theta', \hat{e})) - c(y^p(\theta', \hat{e}), \theta), \quad \forall \theta, \theta' \in \{l, h\}, \tag{4.36}$$

so the private owner maximizes his payoff by choosing $y^p(\theta, \hat{e})$ in state θ. Given that the production level and the subsidy are the same as under the optimal mechanism the payoff of the government has to be the same as well. Q.E.D.

However, if the production technology allows for less commitment some efficiency improving renegotiation cannot be avoided. In a multiperiod context this leads to the unravelling of private information along the lines of the Coase conjecture.[19] The more flexible the production technology the less the government can "price discriminate" between private owners with different costs and the less ex post

[19] See e.g. Gul, Sonnenschein and Wilson (1986).

inefficiencies can be sustained.[20] This is good news if the firm was privatized to an owner-manager. Less price discrimination means a bigger information rent which gives better investment incentives. However, it is bad news in the case of a rentier-owner because less distortion of production means a softer budget constraint for the manager and gives less incentives to invest.

[20] Because the payoff of the government in period 2 increases the more it is able to price discriminate, it will try to condition the subsidy scheme such that the commitment not to renegotiate is maximized. For example, the subsidy could also be conditioned on the employment level or other variables if these were more expensive to change.

4.6. Appendix

Proof of Proposition 4.1:

To simplify notation denote $y(\theta) = y^p(\theta, \hat{e})$, $s(\theta) = s^p(\theta, \hat{e})$ and $q = q(\hat{e})$. The following nonlinear program has to be solved:

$$\max_{y(\theta),s(\theta)} \{q \cdot [b(y(l)) - s(l)] \ + \ (1-q) \cdot [b(y(h)) - s(h)]\} \tag{4.37}$$

subject to

$$-s(l) + c(y(l), l) + s(h) - c(y(h), l) \ \leq \ 0 \tag{4.38}$$

$$-s(h) + c(y(h), h) + s(l) - c(y(l), h) \ \leq \ 0 \tag{4.39}$$

$$-s(l) + c(y(l), l) \ \leq \ 0 \tag{4.40}$$

$$-s(h) + c(y(h), h) \ \leq \ 0 \tag{4.41}$$

$$y(l), \ y(h), \ s(l), \ s(h) \ \geq \ 0 \tag{4.42}$$

Note first that (4.40) can be dropped because it is implied by (4.38) and (4.41):

$$-s(l) + c(y(l), l) \ \leq \ -s(h) + c(y(h), l) \ \leq \ -s(h) + c(y(h), h) \ \leq \ 0\ . \tag{4.43}$$

Let $\mathbf{x} = \{y(l), y(h), s(l), s(h)\}$ and $\lambda = \{\lambda_1, \lambda_2, \lambda_3\}$ and define the Lagrangian function as

$$\begin{aligned} L(\mathbf{x}, \lambda) \ &= \ q \cdot [b(y(l)) - s(l)] \ + \ (1-q) \cdot [b(y(h)) - s(h)] \\ &+ \ \lambda_1 \cdot [s(l) - c(y(l), l) - s(h) + c(y(h), l)] \\ &+ \ \lambda_2 \cdot [s(h) - c(y(h), h) - s(l) + c(y(l), h)] \\ &+ \ \lambda_3 \cdot [s(h) - c(y(h), h)] \end{aligned} \tag{4.44}$$

The problem is that constraints (4.38) and (4.39) are not convex. However, according to the Kuhn-Tucker theorem, if the vector $(\mathbf{x}^*, \lambda^*)$ is a saddle point of the Lagrangian, maximizing it relative to all nonnegative instruments $\mathbf{x}$ and minimizing it relative to all nonnegative Lagrange multipliers λ, i.e. if

$$L(\mathbf{x}, \lambda^*) \ \leq \ L(\mathbf{x}^*, \lambda^*) \ \leq \ L(\mathbf{x}^*, \lambda) \qquad \forall \mathbf{x} \geq 0, \ \lambda^* \geq 0\ , \tag{4.45}$$

then $(\mathbf{x}^*, \lambda^*)$ solves the nonlinear programming program. The Kuhn-Tucker conditions are the first order conditions for a saddle point of $L(\mathbf{x}, \lambda)$. We first show that they uniquely define the solution given in Proposition 4.1. They include (4.38) - (4.42) and:

$$qb_y(y(l)) - \lambda_1 c_y(y(l), l) + \lambda_2 c_y(y(l), h) \ \leq \ 0 \tag{4.46}$$

$$(1-q)b_y(y(h)) + \lambda_1 c_y(y(h), l) - \lambda_2 c_y(y(h), h) - \lambda_3 c_y(y(h), h) \ \leq \ 0 \tag{4.47}$$

$$-q + \lambda_1 - \lambda_2 \ \leq \ 0 \tag{4.48}$$

$$-(1-q) - \lambda_1 + \lambda_2 + \lambda_3 \ \leq \ 0 \tag{4.49}$$

$$\lambda_1, \ \lambda_2, \ \lambda_3 \ \geq \ 0 \ . \tag{4.50}$$

It cannot be optimal to choose $y(l) = y(h) = 0$. This would yield a payoff of at most 0 while the government can obtain strictly more by choosing $y(l) = y^*(l)$, $y(h) = 0$, $s(l) = c(y^*(l), l)$ and $s(h) = 0$. However, if $y(l) = y(h) = 0$ is not the optimal solution to the program, it cannot be a saddle point of the Lagrangian. Suppose (4.48) holds with strict inequality, so $\lambda_1 < q + \lambda_2$. The complementary slackness condition and (4.40) imply that $s(l) = 0$ and $y(l) = 0$. Substituting these in (4.46) gives

$$q \cdot [b_y(0) - c_y(0, l)] \ + \ \lambda_2 \cdot [(c_y(0, h) - c_y(0, l)] \ < 0 \ , \tag{4.51}$$

which is a contradiction to Assumptions (2.3) and (2.8). So (4.48) must hold with equality and

$$\lambda_1 \ = \ q + \lambda_2 \ > \ 0 \ . \tag{4.52}$$

Now we have to distinguish two cases:

(a) $y(h) > 0$, which implies $s(h) > 0$. Complementary slackness implies that (4.49) holds with equality, so $\lambda_3 = 1$. In this case (4.38) and (4.41) are binding and we get

$$s(h) \ = \ c(y(h), h) \ , \tag{4.53}$$

$$s(l) \ = \ c(y(l), l) + c(y(h), h) - c(y(h), l) \ . \tag{4.54}$$

Substituting (4.53) and (4.54) in (4.39) yields

$$[c(y(h),l) - c(y(l),l)] \; - \; [c(y(h),h) - c(y(l),h)] \; \geq \; 0 \; . \tag{4.55}$$

By Assumption (2.8) marginal costs are increasing in θ, so $y(h) \leq y(l)$. Note, $y(h) = y(l) > 0$ is impossible. To see this substitute $\lambda_1 = \lambda_2 + q$ and $\lambda_3 = 1$ in (4.46) and (4.47), note that (4.46) and (4.47) must hold with equality if $y(h), y(l) > 0$, and eliminate $b_y(y)$ to get

$$c_y(y,l) = c_y(y,h) \; , \tag{4.56}$$

a contradiction to Assumption (2.8). Therefore $y(l) > y(h)$. Considering again (4.55) this implies that (4.39) cannot be binding, so $\lambda_2 = 0$ and $\lambda_1 = q$. It now follows from (4.46) and (4.47) that

$$b_y(y(l)) \; = \; c_y(y(l),l), \tag{4.57}$$

$$b_y(y(h)) \; = \; c_y(y(h),h) \; + \; \frac{q}{1-q} \cdot [c_y(y(h),h) - c_y(y(h),l)] \; . \tag{4.58}$$

(b) $y(h) = 0$ and $s(h) = 0$. Since $y(h) = y(l) = 0$ cannot be optimal, $y(l) > 0$ and (4.46) must hold with equality. Furthermore, since (4.38) is binding we get

$$s(l) \; = \; c(y(l),l) \; . \tag{4.59}$$

But then (4.39) cannot be binding, so $\lambda_2 = 0$ which implies $\lambda_1 = q$. Substitution in (4.46) yields again (4.57). So the optimal level of $y(l)$ is the same as in case (a).

We now have to check whether $y(h) = 0$ or $y(h) > 0$ is optimal. The latter is the case iff

$$q \cdot [b(y(l)) - c(y(l),l) - c(y(h),h) + c(y(h),l)] \; + \; (1-q) \cdot [b(y(h)) - c(y(h),h)]$$
$$\geq q \cdot [b(y(l)) - c(y(l),l)] \; , \tag{4.60}$$

or, equivalently, iff

$$b(y(h)) - c(y(h),h) - \frac{q}{1-q} \cdot [c(y(h),h) - c(y(h),l)] \ \leq\ 0\ , \tag{4.61}$$

which is the condition for an interior solution given in Proposition 4.1.

It is straightforward to check for both possible cases that $\lambda^* = \{p, 0, 1\}$ minimizes $L(\mathbf{x}^*, \lambda)$ given $\mathbf{x}^*$. To prove that $\mathbf{x}^*$ as characterized in Proposition 4.1 indeed maximizes $L(\mathbf{x}, \lambda^*)$ we still have to show that $L(\mathbf{x}, \lambda^*)$ is concave, i.e. that the Hessian matrix of $L(\mathbf{x}, \lambda^*)$ is *everywhere* negative semidefinite. To see this note that because of Assumptions (2.2), (2.6) and (2.9)

$$L_{11} = q \cdot [b_{yy}(y(l)) - c_{yy}(y(l),l)] < 0, \tag{4.62}$$

$$\begin{aligned} L_{22} &= (1-q)b_{yy}(y(h)) + q \cdot c_{yy}(y(h),l) - c_{yy}(y(h),h) \\ &= (1-q)[b_{yy}(y(h)) - c_{yy}(c(h),h)] - q[c_{yy}(y(h),h) - c_{yy}(c(h),l)] \\ &< 0\ , \end{aligned} \tag{4.63}$$

while all other second derivatives of L vanish. This establishes the first part of Proposition 4.1.

Finally we have to show that $y^p(h,\hat{e})$ is decreasing in $\hat{e}$. Applying the implicit function theorem we get:

$$\frac{dy^p(h,\hat{e})}{d\hat{e}} = \frac{-\ \frac{-q_e(1-q)-q_e q}{(1-q)^2} \cdot [c_y(y(h),h) - c_y(y(h),l)]}{b_{yy}(y(h)) - c_{yy}(y(h),h) - \frac{q}{1-q} \cdot [c_{yy}(y(h),h) - c_{yy}(y(h),l)]} \ <\ 0. \tag{4.64}$$

Note that if $y(h) > 0$ then $y^p(h,\hat{e})$ is strictly decreasing. Q.E.D.

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Curriculum Vitae

16.06.1961	geboren in Koblenz als Sohn von Dr. Alfred Schmidt und Ursula Schmidt, geb. Klaßen
Juni 1980	Abitur am Werner-Heisenberg-Gymnasium in Neuwied
Okt. 1980 - Jan. 1982	Zivildienst als Erziehungshelfer im Evang. Kinder- und Jugendheim Oberbieber (Neuwied)
April 1982 - Nov. 1987	Förderung durch die Studienstiftung des deutschen Volkes
April 1982 - Sep. 1983	Grundstudium der Volkswirtschaftslehre und der Politikwissenschaft an der Philipps-Universität in Marburg
Okt. 1983	Fortsetzung des Hauptstudiums an der Universität Hamburg
Nov. 1987	Abschluß der Diplomprüfungen in Volkswirtschaftslehre und in Politikwissenschaft an der Universität Hamburg
Okt. 1987	Eintritt in das European Doctoral Program in Quantitative Economics und Beginn des Doktorandenstudiums an der Universität Bonn
Sep. 1988 - Juli 1989	Studium an der London School of Economics (Stipendium des DAAD)
ab Juli 1989	Wissenschaftlicher Mitarbeiter in der wirtschaftspolitischen Abteilung des Instituts für Wirtschafts- und Gesellschaftswissenschaften an der Universität Bonn bei Herrn Professor Schweizer
März/April 1990	Forschungsaufenthalt an der Universität von Tel Aviv auf Einladung von Herrn Professor Rubinstein.