Option contracts and renegotiation: a solution to the hold-up problem

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In this article, we analyze the canonical hold-up model of Hart and Moore under the assumption that the courts can verify delivery of the good by the seller. It is shown that no further renegotiation design is necessary to achieve the first best: simple option contracts, which give the seller the right to take the delivery decision and specify payments depending on whether delivery takes place, allow implementation of efficient investment decisions and efficient trade.

1. Introduction

In a seminal article, Hart and Moore (1988) considered a buyer-seller relationship with observable but unverifiable investment decisions. They argued that contractual incompleteness, due to nonverifiability of the relevant state of the world, combined with the parties' inability to prevent ex post renegotiation will lead to underinvestment in such a classical hold-up problem. This result has attracted considerable attention because it seems to provide a theoretical foundation for the rapidly growing literature on incomplete contracts, which tries to explain economic institutions, such as the allocation of ownership rights or the financial structure of the firm, as second-best solutions to incentive problems in a world in which comprehensive contracts cannot be written.

In this article, we argue that the underinvestment problem in the Hart-Moore model can be overcome if the parties can write simple option contracts. An option contract gives the seller the right (but not the obligation) to deliver a fixed quantity of the good and makes the buyer's contractual payment contingent on the seller's delivery decision. Note that an option contract is feasible only if it is possible to enforce payments conditional on the seller's delivery decision, that is, the court must be able to observe whether the seller...
delivered the good to the buyer. This possibility is explicitly ruled out by Hart and Moore who assume that, if trade fails, a court cannot distinguish whether the seller refused to supply or whether the buyer refused to take delivery. It is this assumption, and only this assumption, from the original Hart–Moore model that we need to abandon in order to achieve the first best.

To put our contribution into perspective, it is useful to relate our results to Aghion, Dewatripont, and Rey (1994). These authors have shown that the underinvestment problem can be solved if renegotiation design is possible, in the sense that the contractual environment allows (i) allocation of all bargaining power in the renegotiation game to one of the contracting parties and (ii) specification of an appropriate default point that obtains if renegotiation breaks down. The logic behind this result is that the party who has all the bargaining power in the renegotiation game becomes residual claimant on total surplus (minus a constant) and thus has the right incentives to invest. Incentives for the other party are then provided through the effect his investment has on the value of the default point. In a second step, the authors present a model of the renegotiation process—quite different from the one assumed by Hart and Moore—that achieves the required renegotiation design through the use of specific performance clauses and penalties for delay in the original contract. 1

In contrast to Aghion, Dewatripont, and Rey, Hart and Moore took the renegotiation process as exogenously given. We shall show that, given their renegotiation process, every option contract results in the allocation of all bargaining power to the buyer in the renegotiation game. Hence, property (i) obtains naturally and the buyer has the right incentives to invest. Although all option contracts result in the same allocation of bargaining power, different option contracts will induce different default points for the renegotiation game. Because the seller has the right to decide whether to deliver, the (implicit) default point for renegotiation is given by whatever delivery decision the seller prefers to make under the terms of the initial contract. We shall show that adjusting the option price (i.e., the additional payment required from the buyer if the seller exercises his option to deliver) provides us with enough flexibility to achieve property (ii) and thus provide the seller with the correct investment incentives. Because we use the same renegotiation game as Hart and Moore, our approach highlights that it is Hart and Moore’s assumption that the court cannot observe whether the seller delivered the good that is crucial to the underinvestment result (and not the exogenously given renegotiation game). 2

In order to further contrast our analysis with Aghion, Dewatripont, and Rey, consider the case in which the good to be traded is indivisible and at most one unit can be traded (which is the only case considered by Hart and Moore and also the focus of most of our article). In this case, Aghion, Dewatripont, and Rey rely on explicit randomization to achieve the first best, whereas no such randomization is necessary with an option contract. Aghion, Dewatripont, and Rey proceed by designing a contract that gives all the bargaining power in their renegotiation game to one party, say, the buyer. The problem then becomes providing the seller with the correct incentives. If no trade is specified as a default point, the seller has no incentive to invest. If trade of one unit is specified as a default point, Chung (1991) also shows that the first best can be achieved if (i) and (ii) are satisfied. However, whereas Aghion, Dewatripont, and Rey offer an explicit contract, which generates a renegotiation game satisfying (i) and (ii), Chung just assumes that these conditions are satisfied.

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2 Remarks to this effect can already be found in Aghion, Dewatripont, and Rey and Hermalin and Katz (1993). Hermalin and Katz do not elaborate the point. Aghion, Dewatripont, and Rey observe that it is possible to allocate all bargaining power to one party in the original Hart–Moore model by choosing the price differential in the original contract appropriately, so that the only element of renegotiation design lacking from their model seems to be the ability to assign a default point different from no trade. This argument is incomplete in that it ignores the fact that, once different default points are introduced in the Hart–Moore model, it may no longer be the case that the price differential influences the distribution of bargaining power as in Hart and Moore. Indeed, the price differential in an option contract has no effect on the distribution of bargaining power. Instead it serves to shift the default point.
point, overinvestment will be induced if the probability that trade is efficient is less than one. To avoid this underinvestment (overinvestment) problem, Aghion, Dewatripont, and Rey propose that the initial contract should specify "trade with probability \( q \)" as a default point, where \( q \) is chosen to provide just the right investment incentives for the seller. Of course, this contract requires that the probability of trade would be enforced by a court if renegotiation fails.

Suppose now that the renegotiation process is the same as in Hart and Moore and consider an option contract that gives the seller the right to supply the good at price \( p_1 \) or not to supply and receive \( p_0 \). In this case, it is always the seller who has to be convinced (through renegotiation) to take the efficient action. The renegotiation game used by Hart and Moore has the property that the buyer can bribe the seller to do the right thing by making him just indifferent between trade and no trade. Thus, the buyer becomes residual claimant on the margin and, as in Aghion, Dewatripont, and Rey, is induced to invest efficiently. What about the seller? There are two possible default points of renegotiation that determine his utility: If the difference between \( p_1 \) and \( p_0 \) is higher than his production cost, the seller will enforce trade. If, however, the difference between \( p_1 \) and \( p_0 \) is smaller than his production cost, he will choose not to trade. Because the seller’s production costs are a random variable, by varying \( p_1 - p_0 \), we can vary the probability of the two default points and give the seller, in expectation, just the right incentives to invest. The main difficulty in showing this result is that, in contrast to Aghion, Dewatripont, and Rey, there is a feedback effect from investment decisions to the (expected) default point induced by an option contract: The seller’s investment affects the distribution of his production costs and thus the probability that "trade at \( p_1 \)" arises as the default point of contract renegotiation.

Although in most of our article we deal with the case in which at most one unit of an indivisible good can be traded, our main results can be generalized to the case in which there are different levels of quantity and/or quality from which to choose. In this case, an option contract specifies one particular specification of the good and a price to be paid if the seller chooses to deliver exactly this specification. If any other specification is delivered (and the contract has not been renegotiated), the buyer is not required to pay more than the base payment, which he would have to pay even if the seller delivered nothing. We provide a simple condition under which the first best can be implemented if such an option contract is enforced by the courts. This condition is automatically satisfied if trade is a zero-one decision. Our result is in stark contrast to the incomplete contracts literature (e.g., Grossman and Hart (1986)) which argues that contracts are incomplete because of the difficulty to specify in advance the good to be traded contingent on a complex state of the world. Our result can be interpreted as showing that a contingent contract is often not necessary but that the first best can be achieved if it is possible to contract on at least one specification.

There is a large and growing recent literature dealing with contractual remedies to the hold-up problem. Rogerson (1992) shows that sequential mechanisms from the implementation literature can be used to achieve the first best under a variety of informational assumptions. However, these mechanisms are typically not renegotiation-proof. MacLeod and Malcolmson (1993) and Edlin and Reichelstein (1993) consider a hold-up problem with a different renegotiation game. They focus on the case in which only one party has to make a relationship-specific investment and show that simple contracts can achieve the first best in this case. For some special cases, their results carry over if both parties have to invest.\(^3\)

\(^3\) MacLeod and Malcolmson (1993) show that the first best can be achieved if (i) investments are not relationship specific but there is a switching cost or (ii) if investments are specific and there is an observable variable that is correlated with the investment levels and that can be contracted upon. Edlin and Reichelstein (1993) can implement the first best if the effect of investments and the effect of the state of the world enter the production costs of the seller (and the valuation of the buyer) in an additively separable manner.
More closely related to our article is the contribution by Hermalin and Katz (1993). These authors consider an environment in which the buyer's valuation and the seller's cost are stochastically independent. They show that a fill-in-the-price contract can achieve the first best in the absence of renegotiation. We show that these fill-in-the-price contracts can easily be embedded in the extensive form of our model in which they correspond to a menu of option contracts from which one party is allowed to choose after costs and benefits have been realized. The particular contract suggested by Hermalin and Katz will indeed not be renegotiated in equilibrium. Whereas our simple contract specifies only two prices to achieve efficient investments and relies on renegotiation to achieve efficient trade, writing a more elaborate fill-in-the-price contract thus avoids renegotiation while still achieving the first best under Hermalin and Katz’s independence assumption.

We organize the remainder of the article as follows. In Section 2, we briefly summarize the model of Hart and Moore and show how the outcome of their renegotiation game is affected if we allow for option contracts. In Section 3, we show that an option contract can achieve the first best. In Section 4, we show that there are interesting cases in which renegotiation never occurs in equilibrium and discuss fill-in-the-price contracts. In Section 5, we extend our main results to the case in which there are different levels of quantity and/or quality of the good from which to choose. In Section 6, we conclude and discuss some further extensions.

2. Description of the model

Consider a buyer and a seller both of whom are risk neutral. At some initial date 0, they can write a contract specifying the terms of trade of one unit of an indivisible good which they may want to exchange at some future date 2. After date 0 but before date 1, the buyer and the seller make relationship-specific investments \( \beta \in [0, \bar{\beta}] \) and \( \sigma \in [0, \bar{\sigma}] \), respectively. These investments are sunk. The buyer’s valuation \( v(\omega^B, \beta) \) and the seller’s production costs \( c(\omega^S, \sigma) \) are determined by their relationship-specific investments and the realization of the state of the world, \( \omega = (\omega^B, \omega^S) \), which is realized at date 1.\(^4\) Let \( \omega \) be distributed on \( \Omega = [0, 1]^2 \) according to the continuous joint density function \( f(\omega) \). The marginal densities are denoted by \( f^B(\omega^B) = \int_0^{\omega^B} f(\omega^B, \omega^S)d\omega^S \) and \( f^S(\omega^S) = \int_0^{\omega^S} f(\omega^B, \omega^S)d\omega^B \).

Let \( h^B(\beta) \) and \( h^S(\sigma) \) denote the strictly increasing and continuous cost functions for the investments. Furthermore, assume that \( v(\omega^B, \beta) \) and \( c(\omega^S, \sigma) \) are continuous in both arguments and strictly positive. Finally, suppose that production costs are nonincreasing in \( \sigma \) for all \( \omega^S \).

Let \( q \in \{0, 1\} \) be the level of trade and \( p \) the (possibly negative) net payment of the buyer to the seller. Then the utilities of the buyer and the seller after date 2 are given by

\[
u^B = q \cdot v(\omega^B, \beta) - p - h^B(\beta) \tag{1}
\]

\[
u^S = p - q \cdot c(\omega^S, \sigma) - h^S(\sigma) \tag{2}
\]

The problem of the parties at date 0 is to design a contract that implements efficient investment and trade decisions, i.e., that maximizes expected total surplus

\[
W(\beta, \sigma) = \int_0^{\bar{\beta}} \int_0^{\bar{\sigma}} [v(\omega^B, \beta) - c(\omega^S, \sigma)]^+ f(\omega)d\omega^Sd\omega^B - h^B(\beta) - h^S(\sigma), \tag{3}
\]

\(^4\) Note that the specification of \( v(\cdot, \beta) \) and \( c(\cdot, \sigma) \) assumes that there are no direct externalities of the investments. However, there is of course an indirect externality because the investments affect the probability of trade. It is this indirect externality that is the focus of Williamson (1985) and Grossman and Hart (1986).
where we shall frequently use the notation $[\cdot]^+ = \max\{0, \cdot\}$ throughout the remainder of this article. Given our continuity assumptions, $W(\beta, \sigma)$ is continuous in $\beta$ and $\sigma$. The boundedness assumption on $\beta$ and $\sigma$ thus implies that the set of maximizers of $W(\cdot, \cdot)$ is always nonempty. Denote by $(\beta^*, \sigma^*)$ a pair of first-best investment levels that maximizes (3). For convenience, we assume that $(\beta^*, \sigma^*)$ is unique. Also, let $Q^*(\omega, \beta, \sigma) = \arg\max_{\alpha}[v(\omega^\beta, \beta) - c(\omega^\delta, \sigma)]$ denote the set of ex post efficient levels of trade.

The first best could easily be achieved if it were possible to contract upon the level of investment. However, we assume that, although investments $\beta$ and $\sigma$ as well as the state of the world $\omega$ (and so $\nu$ and $c$) are perfectly observable by both agents, they cannot be verified to any third party, e.g., the courts. Thus, the contract cannot enforce outcomes contingent on these variables.

Trade takes place ($q = 1$) if and only if the seller delivers the good at date 2 and the buyer accepts delivery. Hart and Moore assume that the courts can only observe whether $q = 0$ or 1, but if $q = 0$, they cannot distinguish whether the seller or the buyer was unwilling to trade. In contrast, we assume that the courts can observe whether the seller delivered the good, $d = 1$, or not, $d = 0$. Thus, in our model, it is possible to write an initial contract signed at date 0 that specifies two different prices ($p_0$, $p_1$) depending on whether $d = 1$ or $d = 0$. The initial contract could, in principle, also be conditional on verifiable messages exchanged between the parties. Because we want to show that a simple option contract implements the first best already, we do not need to consider these more complicated mechanisms.

After date 1, the initial contract can be renegotiated. To simplify the proof of the following result, we assume that there is only one point in time between dates 1 and 2 at which the parties can send signed contract offers ($p^i_0$, $p^i_1$), $i = S, B$, to each other. After trade decisions have been made at date 2, the parties can decide simultaneously whether to present any renegotiation offers they have received to the court. The court can observe delivery and will enforce payments as specified in the initial contract unless

(a) exactly one party has produced a contract signed by the other party that specifies different terms of trade, or

(b) both parties produced identical contracts signed by the other party and specifying different terms of trade,

in which cases the payments of the new contract(s) are enforced.

As Hart and Moore, we are interested in the case in which renegotiation is costless. However, if sending renegotiation offers is costless, the renegotiation subgame that occurs after date 1 may have multiple subgame-perfect equilibrium outcomes. To obtain Proposition 1, which summarizes the outcome of the renegotiation game after an option contract ($p_0$, $p_1$) as defined above has been signed, we thus focus on the subgame-perfect equilibrium strategies in which an agent makes a renegotiation offer only if doing so strictly increases his expected payoff. This result is the counterpart to Proposition 1 of Hart and Moore.

**Proposition 1.** Let ($p_0$, $p_1$) be the initial option contract signed at date 0. Given investment levels $\beta \in [0, \hat{\beta}]$ and $\sigma \in [0, \check{\sigma}]$, the traded quantity satisfies $q \in Q^*(\omega, \beta, \sigma)$ and the payment of the buyer to the seller is given by

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5 Hart and Moore allow for a finite number of renegotiation dates and arbitrarily complex contract offers. However, they do not properly specify strategy spaces. In order to have a well-defined game and to keep the exposition self-contained, we consider a simplified renegotiation process. Alternatively, the same arguments used by Hart and Moore to support their Proposition 1 could be used to obtain our Proposition 1. See Appendix A of their article.
(i) if \( p_1 - p_0 \leq c(\omega^*, \sigma) \), then \( p = p_0 + q \cdot c(\omega^*, \sigma) \)
(ii) if \( p_1 - p_0 > c(\omega^*, \sigma) \), then \( p = p_1 - c(\omega^*, \sigma) + q \cdot c(\omega^*, \sigma) \).

Proof. See Appendix A.

Although the formal proof is relegated to Appendix A, the basic intuition for this result is easy to understand and will be explained in the remainder of this section.\(^6\)

Given the initial contract with prices \( p_0 \) and \( p_1 \), the seller is willing to trade if and only if \( p_1 - p_0 > c \). If the seller chooses \( d = 0 \), then \( q = 0 \) follows automatically. If he chooses \( d = 1 \), then it is a dominant strategy for the buyer to accept delivery (because \( v > 0 \) and the payment of the buyer is independent of whether he accepts delivery), so \( q = 1 \). Suppose the privately optimal decision of the seller is also socially optimal. In these cases, there is no scope for renegotiation: Efficient trade decisions will already result from the original contract, and each player can guarantee himself the corresponding payoff by not making a renegotiation offer and withholding any offer he might have received.

However, if the seller’s privately optimal delivery decision is not efficient, there is scope for renegotiation. Note that renegotiation can only succeed if the buyer offers a new contract. To see this, suppose the buyer made no offer at the renegotiation stage. Then, no matter what new contract has been sent by the seller, the buyer can always induce the courts to enforce the old contract \((p_0, p_1)\) by withholding any renegotiation offer he received. Therefore, the seller will not make the efficient trading decision until he has a new contract in hand, offered and signed by the buyer, which guarantees him at least what he could get from sticking to the old contract and taking the inefficient action. Thus, the buyer must give in and adjust prices such that they are more favorable for the seller if he makes the efficient delivery decision. On the other hand, the buyer need not give in too much. He makes the renegotiation offer, so he can suggest new prices that make the seller just indifferent whether to reverse his delivery decision. Hence, the buyer has all the bargaining power in the renegotiation game. In case (i), if \( v > c \), the buyer thus needs to raise the delivery price to \( p_0 + c \) to induce the seller to produce and deliver. Note that, because \( v > c \), it is profitable for the buyer to do so instead of forgoing delivery under the original contract. By the same argument, the buyer needs to raise the no-delivery payment to \( p_1 - c \) in case (ii), provided that trade is inefficient \((v < c)\), in order to induce the seller to forgo production; because \( v < c \), he will choose to do so.

Let us finally compare Proposition 1 with the corresponding Proposition 1 in Hart and Moore. They assume that the courts cannot observe delivery \((d)\) but only whether trade took place \((q)\). Thus, a Hart-Moore contract also consists of two prices \( p_0 \) and \( p_1 \), but now \( p_i \) is the payment if \( q = i \), \( i \in \{1, 2\} \). The analysis is very similar except for the following two cases:

(a) If \( v < c < p_1 - p_0 \), a Hart-Moore contract is not renegotiated and yields \( q = 0 \) and \( p = p_0 \). Renegotiation is not necessary because the buyer, who does not want to trade, can prevent inefficient trade unilaterally, guaranteeing himself \( u^b = -p_0 \geq c - p_1 \).
(b) If \( p_1 - p_0 > v > c \), a Hart-Moore contract is renegotiated, whereas an option contract is not. Trade is efficient, but without renegotiation of the Hart-Moore contract, the buyer would veto trade. Thus, in equilibrium, the seller has to offer to lower the trade payment to \( p = v + p_0 \), and payoffs are \( u^b = v + p_0 - c \) and \( u^o = -p_0 \).

3. Efficient option contracts

What investment incentives are given by an option contract? Using Proposition 1, we can derive the expected utilities of the parties as a function of their investment choices

\(^6\) Throughout the following heuristic discussion, we shall ignore cases in which either \( p_1 - p_0 = c(\omega^*, \sigma) \) or \( v(\omega^*, \beta) = c(\omega^*, \sigma) \).
and the initial contract. To do so, it will be convenient to choose a slightly different parameterization of an option contract, namely, to identify an option contract with a pair \((p_0, k)\), where \(p_0\) is a base payment, which has to be made anyway, and \(k = p_1 - p_0\) is the option price. Denote the expected utility of agent \(i\) by \(U'(\sigma, \beta, p_0, k)\). We then have

**Corollary 1.** The expected utilities of the agents are given by

- For the buyer, we have
  \[
  U^B(\omega^B, \beta, p_0, k) = -h^B(\beta) - p_0 + \int_0^1 \int_0^1 [v(\omega^B, \beta) - c(\omega^S, \sigma)]^+ f(\omega^B) d\omega^B d\omega^S
  \]
  \[
  - \int_0^1 [k - c(\omega^S, \sigma)]^+ f^S(\omega^S) d\omega^S
  \]
  \[\text{(4)}\]

- For the seller, we have
  \[
  U^S(\sigma, \beta, p_0, k) = -h^S(\sigma) + p_0 + \int_0^1 [k - c(\omega^S, \sigma)]^+ f^S(\omega^S) d\omega^S.
  \]
  \[\text{(5)}\]

**Proof.** Using Proposition 1, the date 2 payoffs of the buyer and the seller are given by

- For the buyer, we have
  \[
  u^B = -h^B(\beta) + \begin{cases} 
  -p_0 & \text{in (i) if } v(\omega^B, \beta) < c(\omega^S, \sigma) \\
  v - p_0 - c & \text{in (i) if } v(\omega^B, \beta) \geq c(\omega^S, \sigma) \\
  c - p_0 - k & \text{in (ii) if } v(\omega^B, \beta) < c(\omega^S, \sigma) \\
  v - p_0 - k & \text{in (ii) if } v(\omega^B, \beta) \geq c(\omega^S, \sigma)
  \end{cases}
  \]
  \[\text{(6)}\]

- For the seller, we have
  \[
  u^S = -h^S(\sigma) + \begin{cases} 
  p_0 & \text{in (i)} \\
  p_0 + k - c & \text{in (ii)}
  \end{cases}
  \]
  \[\text{(7)}\]

Integrating over \(\omega^B, \omega^S\) yields (4) and (5). \(Q.E.D.\)

Note that, for any option contract, the buyer’s payoff is simply total surplus minus an expression that does not depend on his investment decision. This fact can be understood by noting that, whenever renegotiation occurs, the renegotiated price is determined by the seller’s cost and thus independent of the buyer’s investment decision. Consequently, the buyer receives the full marginal return on his investment if and only if trade is efficient. Hence, given the investment choice of the seller, the buyer will always invest efficiently. The problem is thus reduced to find an option contract that induces the seller to choose the efficient investment level \(\sigma^*\).

Given an option contract \((p_0, k)\), the seller chooses \(\sigma\) to solve

\[
\max_{\sigma} U^S(p_0, k, \sigma) = -h^S(\sigma) + p_0 + \int_0^1 [k - c(\omega^S, \sigma)]^+ f^S(\omega^S) d\omega^S,
\]

where we dropped \(\beta\) as an argument of his utility function because his payoff is independent of the buyer’s investment decision. Note that the set of maximizers of (8) is nonempty for every given option contract and depends only on the option price \(k\). We let \(\Sigma(k)\) denote the set of maximizers of (8) for a given \(k\).

Lemma 1 shows that there always exist option prices \(k\) and \(\tilde{k}\), \(k < \tilde{k}\), such that the seller is induced to underinvest (overinvest) relative to the efficient level \(\sigma^*\). The intuition for this result can be seen from Corollary 1: If a sufficiently small option price has been chosen, then \(k - c(\omega^S, \sigma)\) = 0 with probability 1 and the seller’s payoff at date 2 (net of investment costs) is simply \(p_0\) and thus independent of his investment level. Consequently, the seller will not invest. On the other hand, if the option price is chosen sufficiently high, then \(k - c(\omega^S, \sigma)\) = \(k - c(\omega^S, \sigma)\) with probability 1. Now the seller's
payoff at date 2 is \( p_1 - c \). Thus, an investment in cost reduction will pay off with probability 1. Because the probability that trade is efficient is less than or equal to 1, such an option price will provide the seller with an incentive to overinvest.

Lemma 1. Let \( \hat{k} = \max_{\omega^s, m} c(\omega^s, \sigma) \). Then

\[
\sigma \in \Sigma(0) \Rightarrow \sigma \leq \sigma^* \\
\sigma \in \Sigma(\hat{k}) \Rightarrow \sigma \geq \sigma^*.
\]

Proof. See Appendix A.

The possibility to induce overinvestment (underinvestment) by varying the option price suggests that it should be possible to find an option price \( k^* \) in \([\hat{k}, \hat{k}]\), which gives the seller the desired incentive to choose his first-best investment level. Whenever the seller's maximization problem for a given option price is sufficiently well behaved, this is indeed the case. In particular, we can now state our main result.

Proposition 2. Suppose the seller's maximization problem (8) has a unique solution \( \sigma(k) \) for all \( k \in [0, \hat{k}] \). Then there exists an option contract \((p_0, k)\), which implements efficient investment and trade decisions. Furthermore, any division of the \textit{ex ante} surplus can be achieved by choosing \( p_0 \) appropriately.

Proof. By Berge's maximum theorem, the function \( \sigma(k) \) is continuous, and by the intermediate value theorem and Lemma 1, it follows that there exists \( k^* \in [0, \hat{k}] \) such that \( \sigma(k^*) = \sigma^* \). Given any option contract \((p_0, p_1)\) with \( p_1 = p_0 + k^* \), the seller will thus choose the efficient investment level \( \sigma^* \). Anticipating this, the unique best response of the buyer is to choose \( p^* \). Hence, every such contract implements first-best investment decisions, and renegotiation yields efficient trade by Proposition 1. Finally, note that we are free to choose the base payment \( p_0 \). Thus, it follows immediately from the expressions in Corollary 1 that any division of the \textit{ex ante} surplus can be achieved. Q.E.D.

The uniqueness assumption in Proposition 2 is essentially a continuity requirement, ensuring that, by varying the option price, it is possible to fine-tune the investment incentives of the seller. Obviously, this assumption is satisfied if the seller's payoff function given by (8) is strictly quasiconcave in \( \sigma \). Note, however, that the standard assumptions that \( h^s(\cdot) \) and \( c(\cdot, \sigma) \) are convex in \( \sigma \) are not sufficient to guarantee this property. The problem is that a variation in \( \sigma \) not only affects \( h^s(\cdot) \) and \( c(\cdot, \cdot) \), but also the set of states of the world in which \( k - c(\omega^s, \sigma) \geq 0 \) and thus the probability that the seller's investment pays off. If for some \((k, \sigma)\) this effect is too strong, it may generate convexities in the seller's utility function. Appendix B discusses explicit conditions on the underlying cost functions \( h^s(\cdot) \) and \( c(\cdot, \cdot) \) that ensure that the seller's problem is strictly concave in \( \sigma \) for all possible option prices.

How are our findings different from those of Hart and Moore? Given a Hart-Moore contract, at least one party will block trade whenever trade is inefficient. Furthermore, if \( c < p_1 - p_0 < v \), trade is efficient and will take place because both parties are willing to trade. In these cases, private and social marginal returns of investments coincide. In all other cases, however, at least one party has an incentive to underinvest: If \( v > c > p_1 - p_0 \), the buyer's incentives are fine but the seller's marginal return is 0, so he will underinvest. If \( p_1 - p_0 > v > c \), an opposite result is obtained. The seller has the right incentives, but the buyer's investment does not pay off. Hart and Moore's underinvestment result stems from the fact that, in general, it is impossible to choose \( p_1 - p_0 \) such that the probabilities of these two cases vanish at the same time. In contrast, given our option contracts, the seller is induced to overinvest if \( v < c < p_1 - p_0 \) and to underinvest if \( v > c > p_1 - p_0 \). By choosing \( k \) appropriately, it is possible to balance the
probabilities of these cases such that, on average, the seller has just the right incentives to invest.

4. Efficiency without renegotiation

Our argument for the efficiency of simple option contracts relies on the assumption that the parties can use costless renegotiation to avoid \textit{ex post} inefficient delivery decisions by the seller. Although this is in the tradition of the contributions by Hart and Moore (1988) and Aghion, Dewatripont, and Rey (1994), the question remains whether there are circumstances in which the terms of the original contract can be designed to induce both efficient investment decisions and \textit{ex post} efficient delivery decisions without renegotiation. Addressing this issue will also allow us to explain how our article relates to the recent work by Hermalin and Katz (1993), which does not use renegotiation to achieve the first best.

Suppose first that, given the optimal investment levels, trade is efficient with probability 1. In this case, the first best can be achieved without renegotiation by choosing a sufficiently high option price.

\textbf{Proposition 3.} Suppose that, given the efficient investment choices \((\beta^*, \sigma^*)\), trade is efficient with probability 1. Then any option contract with \(k = \tilde{k}\) implements efficient investment decisions and the initial contract is renegotiated with probability 0.

\textit{Proof.} See Appendix A.

The result in Proposition 3 is stronger than the corresponding “no renegotiation” result in Hart and Moore, which not only requires that trade be efficient with probability 1 but also that one find a constant \(k\) such that, with probability 1, \(\nu(\omega^\beta, \beta^*) \geq k \geq c(\omega^\beta, \sigma^*)\). The reason why this additional condition appears in their result but not in ours is simple. To ensure that an option contract is not renegotiated, it suffices to ensure that the seller always prefers to deliver under the original contract. This can be done by choosing \(k\) sufficiently large. With a Hart–Moore contract, on the other hand, the buyer has the power to veto trade under the original contract. Thus, to avoid renegotiation, it is also necessary to ensure that the buyer wants to trade under the original contract, which requires \(k \leq \nu(\omega^\beta, \beta^*)\) for all \(\omega^\beta\).

A different approach to avoid renegotiation (which applies more generally) is to specify a more complicated initial contract that requires one party to send a verifiable message to the other party after the uncertainty has been resolved. This is the approach suggested by Hermalin and Katz (1993). Under the additional assumption that the seller’s cost and the buyer’s valuation are stochastically independent, these authors show that fill-in-the-price contracts can implement the first best. Their idea is easily embedded in our extensive form: Suppose that the initial contract specifies that, after date 1, the buyer has to announce an option price \(k \in \mathbb{R}\). This announcement can be verified by the court. The initial contract also specifies a real-valued function \(p_0(k)\) with the interpretation that \((p_0(k), k)\) is the option contract in force if the buyer announces the option price \(k\).

Suppose that renegotiation of the option contract selected by the buyer is not feasible. Under this condition, Proposition 1 in Hermalin and Katz shows that, if the initial contract specifies the menu of option contracts given by

\[ p_0(k) = \int_0^1 \left[ \nu(\omega^\beta, \beta^*) - k \right] + f^\beta(\omega^\beta) d\omega^\beta + t, \tag{9} \]

where \(t\) is an arbitrary constant, then the buyer will select the option price \(k = c(\omega^\delta, \sigma)\) and the seller will take the efficient delivery decision in equilibrium. Furthermore, this
contract induces efficient investment decisions. To see this, note that, because
\[ k = c(\omega^s, \sigma), \]
the seller’s expected payoff is given by
\[ U^s(\sigma) = \int_0^1 p_0(c(\omega^s, \sigma)) f^s(\omega^s) d\omega^s + t \]
\[ = \int_0^1 \left[ \int_0^1 \left[ \nu(\omega^b, \beta^*) - c(\omega^s, \sigma) \right] f^b(\omega^b) d\omega^b \right] f^s(\omega^s) d\omega^s + t. \] (10)

If \( \omega^b \) and \( \omega^s \) are stochastically independent, this expression equals the expected social surplus (given \( \beta^* \)) as a function of \( \sigma \), so the seller has just the right incentives to invest.\(^7\)

On the other hand, by setting the option price \( k \) equal to the seller’s cost, the buyer extracts all the surplus from the seller (minus the constant \( p_0(k) \), which is independent of the buyer’s investment). Hence, given that the seller chooses \( W^* \), the buyer is residual claimant of social surplus on the margin and will also invest efficiently.

Let us now allow for renegotiation of the option contract selected by the buyer. That is, suppose that, after the buyer has made his selection from the menu of option contracts, the parties are free to renegotiate the resulting contract, as in Section 2. Clearly, this will not affect Hermelin and Katz’s argument if it is the case that the buyer will still find it optimal to select the option price \( k = c(\omega^s, \sigma) \) because then \( (p_0(k), k) \) induces efficient trade and there is nothing to be renegotiated. As the following result shows, this is indeed the case.

Proposition 4. Suppose the initial contract specifies the menu of option contracts given by (9). Then for all \( \omega \) there is an equilibrium in which the buyer selects the option price \( k = c(\omega^s, \sigma) \) and the resulting contract is not renegotiated.

Proof. See Appendix A.

This result shows that an initial contract as specified in (9) implements an efficient allocation without renegotiation if the buyer’s valuation and the seller’s cost are stochastically independent. The tradeoff, however, is that the parties have to specify a more complicated menu of contracts initially.

5. Variable quantities and/or qualities

In this section, we discuss briefly a simple extension of our main result to the case where \( q \in Q \) and \( Q \) is some (finite or infinite) subset of an Euclidean space of possible quantities and/or qualities of the good to be produced and consumed. We shall derive a simple condition, which is necessary for an option contract to implement the first best. This condition is automatically satisfied if \( q \in \{0, 1\} \). Assuming (as in Proposition 2) that the seller’s maximization problem has a unique solution, this condition is also sufficient to guarantee implementation of an efficient allocation.

The case discussed in this section is a strict generalization of the case considered in Sections 2 and 3. In the general case, an option contract specifies some level of quantity and/or quality \( q_i \) and two prices, \( p_0 \) and \( p_1 \). We assume that the court can distinguish whether \( q_i \) was delivered. The contract says that, if the seller delivers \( q_i \), the buyer is required to pay the price \( p_1 \). If any other \( q \neq q_i \) is delivered and if the contract has not been renegotiated, then the buyer can keep \( q \) and must pay only \( p_0 \). In the renegotiation game, each agent may propose a new contract \( (\bar{p}_0, \bar{p}, \bar{q}) \) specifying some \( \bar{q} \), a price \( \bar{p} \) if

\(^7\) If \( \omega^b \) and \( \omega^s \) are not independent, there is no obvious way to design a fill-in-the-price contract along the lines of Hermelin and Katz such that both parties have the right incentives to invest.
\( \bar{q} \) is delivered, and a no-trade payment \( \bar{p}_0 \). The utilities of the buyer and the seller are given by

\[
\begin{align*}
  u^B &= v(q, \omega^B, \beta) - p - h^B(\beta) \\
  u^S &= p - c(q, \omega^S, \sigma) - h^S(\sigma).
\end{align*}
\]

(11) (12)

Trade of \( q \) takes place if and only if the seller delivers \( q \) at date 2 and the buyer accepts delivery. Let \( q_0 \in Q \) denote the event of no trade with \( v(q_0, \cdot) = c(q_0, \cdot) = 0 \). For all \( q \neq q_0 \), the valuation of the buyer and the production cost of the seller are strictly positive. As before, we also assume that production costs are nonincreasing in \( \sigma \) and continuous.

The first-best investment levels maximize

\[
W(\beta, \sigma) = \int_0^1 \int_0^1 \left[ \max_{\omega \in \tilde{Q}} \{ v(q, \omega^B, \beta) - c(q, \omega^S, \sigma) \} \right] f(\omega) d\omega^B d\omega^S - h^B(\beta) - h^S(\sigma).
\]

(13)

Assume that there exists a unique pair \((\beta^*, \sigma^*)\) maximizing this expression, and let \( Q^*(\omega, \beta, \sigma) = \arg \max_{q \in \tilde{Q}} \{ v(q, \omega^B, \beta) - c(q, \omega^S, \sigma) \} \) be nonempty. The following proposition summarizes the outcome of the renegotiation game and is the counterpart of Proposition 1:

Proposition 5. Let \((p_0, p_1, q_1)\) be the initial option contract signed at date 0. Given investment levels \( \beta \in [0, \beta] \) and \( \sigma \in [0, \sigma] \), the specifications of trade satisfies \( q \in Q^*(\omega, \beta, \sigma) \) and the payment from the buyer to the seller is given by

(i) if \( p_1 - p_0 \leq c(q_1, \omega^S, \sigma) \), then \( p = p_0 + c(q_1, \omega^S, \sigma) \)

(ii) if \( p_1 - p_0 > c(q_1, \omega^S, \sigma) \), then \( p = p_1 + c(q_1, \omega^S, \sigma) - c(q_1, \omega^S, \sigma) \).

The formal proof is omitted because it is a simple generalization of the proof of Proposition 1. To give some intuition for it, consider two cases in turn.

(i) If \( p_1 - p_0 < c(q_1, \omega, \sigma) \), then in the absence of renegotiation, the seller will refuse to trade \((q = q_0)\). Clearly, delivering \( q_1 \) is not profitable. Furthermore, it is never profitable for the seller to deliver any other \( \bar{q} \neq q_1 \) because production costs are positive, whereas the payment \( p_0 \) is independent of whether he delivers \( \bar{q} \) or refuses to trade. If \( q_0 \in Q^*(\omega, \beta, \sigma) \), i.e., no trade is efficient, then there is no scope for renegotiation and the outcome is given by \((q_0, p_0)\). So suppose that \( q_0 \notin Q^* \). The seller is only willing to deliver \( \bar{q} \neq q_0 \) if he gets a renegotiation offer signed by the buyer saying that \( \bar{q} \) will be traded for payment \( \bar{p} \), where \( \bar{p} \) has to be large enough to give the seller at least the utility of his default point \((q_0, p_0)\). Hence, the buyer’s renegotiation offer will satisfy

\[
\bar{p} - c(\bar{q}, \omega^S, \sigma) = p_0
\]

(14)

and the seller will accept this contract, deliver \( \bar{q} \), and enforce \( \bar{p} \). Because the buyer

---

* Although an option contract unambiguously specifies what payments should be enforced by the courts, it could be argued that such a contract is unlikely to be enforceable in practice. In particular, if the seller chooses a specification \( q \) that departs only slightly from the \( q_1 \) agreed upon in the contract, the price drops from \( p_0 \) to \( p_e \) even if the utility loss incurred by the buyer is small. However, when the courts feel that damage payments depart too much from actual (or expected) damages, they may dismiss them as inadequate or punitive and refuse to enforce them. See Edlin and Reichelstein (1993) and the literature cited there. This problem is common to most theoretical analyses of contracts. Note, however, that the general logic of our arguments applies even if the courts enforce \( p_1 \) for any \( q \) delivered by the seller, which is in a neighborhood of \( q_1 \), as long as this implicit option contract induces overinvestment. We are grateful to Mike Riordan for this observation.
can extract all the surplus from the seller, the optimal renegotiation offer satisfies 
\( \hat{q} \in Q^*(\omega, \beta, \sigma) \).

(ii) If \( p_1 - p_0 > c(q_1, \omega, \sigma) \), the default point of the seller is to deliver \( q_1 \). If \( q_1 \) is the efficient level of trade, there is nothing to renegotiate, so suppose that 
\( q_1 \notin Q^*(\omega, \beta, \sigma) \). Again, the buyer has to make an offer that induces the seller not to deliver \( q_1 \). Thus, in equilibrium, the buyer will offer \( (\tilde{q}, \tilde{p}) \) such that

\[
\tilde{p} - c(\tilde{q}, \omega^s, \sigma) = p_1 - c(q_1, \omega^s, \sigma)
\]

and \( \hat{q} \in Q^*(\omega, \beta, \sigma) \). Again, the seller will accept this offer, deliver \( \tilde{q} \), and enforce \( \tilde{p} \).

Anticipating this renegotiation outcome, the expected utilities of the agents are given by

\[
U^b(\sigma, \beta, p_0, k, q_1) = \int_0^1 \int_0^1 \max_{\omega \in Q} [v(q, \omega^s, \beta) - c(q, \omega^s, \sigma)] f(\omega)d\omega^s d\omega^s
- h^b(\beta) - p_0 - \int_0^1 [k - c(q_1, \omega^s, \sigma)]^+ f^s(\omega^s)d\omega^s
\]

\[
U^s(\sigma, \beta, p_0, k, q_1) = -h^s(\sigma) + p_0 + \int_0^1 [k - c(q_1, \omega^s, \sigma)]^+ f^s(\omega^s)d\omega^s.
\]

For any option contract, the buyer's expected payoff coincides with social welfare minus a term that is independent of his investment decision. Thus, the buyer will always invest efficiently. The investment incentives of the seller depend on the choice of \( q_1 \) and \( k \), and we let \( \Sigma(q_1, k) \) denote the set of maximizers of (17) given these parameters. Also, let \( k(q_1) = \max_{\omega \in Q} c(q_1, \omega^s, \sigma) \).

The crucial consideration in determining whether it is possible to achieve the first best with an option contract is whether it is possible to induce the seller to invest at least the efficient amount by specifying a sufficiently high option price. In Lemma 2, we give a necessary and sufficient condition for this to be the case. The condition requires that there exists a \( q_1 \) such that the seller is induced to overinvest if he receives the returns for his investment with probability 1. Note that, if \( Q = \{0, 1\} \), this condition is always satisfied, as was shown by Lemma 1.

Lemma 2. There exists an option contract that induces the seller to invest at least the efficient amount, i.e., \( \exists(q_1, k): \sigma^* \leq \max \Sigma(q_1, k) \), if and only if there exists \( q_1 \in Q \) such that

\[
\exists \sigma \geq \sigma^*: \sigma \in \arg\max_{\sigma'} \int_0^1 - c(q_1, \omega^s, \sigma') f^s(\omega^s)d\omega^s - h^s(\sigma').
\]

In particular, the first best cannot be implemented if (18) fails for all \( q_1 \).

Proof. See Appendix A.

The economic interpretation of (18) will be discussed after Proposition 6, which is the counterpart to Proposition 2.

\[\text{As in Proposition 2, the following result imposes a uniqueness requirement on the solution to the seller's maximization problem. Note that this assumption is only required for one particular choice of } q_1. \text{ Specifically, it suffices to find a specification } q_1 \text{ such that (18) holds and such that } c(q_1, \cdot, \cdot) \text{ satisfies the conditions discussed in Appendix B.}\]
Proposition 6. Suppose there exists a \( q_i \) satisfying (18). Furthermore, assume that, given this \( q_i \), there exists a unique \( \sigma(k) \) maximizing the seller’s payoff function (17) for all \( k \in [0, \tilde{k}(q_i)] \). Then there exists an option contract \((p_0, p_1, q_i)\) that implements efficient investment and trade decisions. Furthermore, any division of the \emph{ex ante} surplus can be achieved by choosing \( p_0 \) appropriately.

\textbf{Proof.} Let \( q_i \) be as specified in the statement of the proposition. Then \( \sigma(0) = 0 \) (cf. the proof of Lemma 1) and \( \sigma(\tilde{k}(q_i)) \geq \sigma^* \) (cf. Lemma 2). The result then follows from the uniqueness assumption on \( \sigma(k) \) as in the proof of Proposition 2. \( Q.E.D. \)

In the literature on incomplete contracts, it has often been claimed that \( q \) is noncontractible \emph{ex ante} because it is too difficult to specify the good in advance, in particular because the optimal specification may depend on a complex realization of the state of the world. The above proposition shows that optimal investment incentives can be given with a simple option contract in which only one specification of the good \((q_i)\) has to be described.\(^{10}\) If this good were traded with probability 1, the seller would be induced to overinvest. By choosing the option price \( k \) appropriately, the incentives to overinvest (if \( q_i \) is the default point) and to underinvest (if \( q_0 \) is the default point) can be balanced such that the seller will invest efficiently, whereas renegotiation ensures efficient trade.

However, a necessary condition for an option contract to implement the first best is that (18) holds. If \( q \) is interpreted as the quantity to be produced, this condition is innocuous. It is natural to assume that the marginal benefit of investment is nondecreasing with the quantity of trade in all states of the world. Thus, it would be sufficient to pick \( q_i \) as the largest \( q \) that is traded with positive probability in the first best in order to induce the seller to overinvest. If \( q \) represents different levels of quality, which can be ordered along the real line such that higher levels of quality make a higher level of investment more desirable in all states of the world, (18) is also unproblematic. However, if \( q \) stands for different specifications of the good and, if for any specification, the productivity of the investment depends on the realization of the state of the world, then there are natural examples in which (18) is violated. As an illustration, suppose that, for any given \( q_i \in Q \), the investment pays off only for some states of the world but is unproductive in others. Thus, if any fixed \( q_i \) is traded with probability 1, only some modest investment is optimal. On the other hand, the \emph{ex post} efficient \( q^* \) depends on the state of the world. Thus, given that \( q^*(\omega) \) will be traded, the investment may now pay off in all states of the world. Hence, the socially optimal investment level \( \sigma^* \) may be higher than the optimal investment level given any fixed \( q_i \in Q \).

A related problem that may prevent the implementation of the first best with a simple option contract is the possibility that investments may be multidimensional. For example, suppose that the seller has a choice between investing in a general purpose technology or in one of several specific purpose technologies that are only useful if a given specification is produced. Because there is uncertainty about the optimal specification \emph{ex ante}, it may be efficient to invest in the general purpose technology, whereas an option contract will provide the seller with a strong incentive to invest in a technology that is tailored to the specification in the option contract. Although the analysis of the hold-up problem with multidimensional investment decisions is beyond the scope of this article, it clearly is an important topic for future research.

\(^{10}\) Note that the incomplete contracts literature typically assumes that it is possible to contractually describe a single specification \emph{ex post}, so it should also be possible to do this \emph{ex ante}. However, it is essential that this specification be described unambiguously, which may be more difficult if the good does not yet exist (e.g., if some research is necessary to develop it).
6. Conclusions

We have shown that, in the canonical hold-up model introduced by Hart and Moore (1988), the first best can be achieved if the courts can verify delivery of the good by the seller. This can be done using a very simple contract that does not rely on renegotiation design or complicated revelation mechanisms. The crucial feature of the contracts we used is that one of the parties can decide unilaterally whether trade takes place. This is why we called them option contracts.

Throughout the article, we restricted attention to the case in which both parties are risk neutral and in which there are only indirect externalities of the investment decisions. This is the case considered in most of the hold-up and incomplete contracts literature. We question whether an option contract can implement the first best in more complex environments in which these assumptions are relaxed. There is no hope that an option contract can allocate risk efficiently. This would require that the default point of renegotiation vary continuously with the realization of the state of the world, which is impossible to achieve with the very simple instrument considered in this article. Option contracts are more successful in dealing with one-sided direct externalities. Suppose that the investment of the seller affects the valuation of the buyer directly, e.g., because it has an impact on the quality of the good. In this case, an option contract can induce both parties to invest efficiently, provided that it is still possible to induce the seller to overinvest. In this case, we can choose the option price such that, in expectation, the seller has just the right incentives to invest. Given that he chooses the first-best level $\sigma^*$, the unique best response of the buyer is to also invest efficiently. On the other hand, if there is a two-sided direct externality, option contracts fail to implement the first best. Although the seller can be given the right incentives as just described, the buyer will not take into account the impact of his investment on the costs of the seller. An interesting question for future research is whether there are other contracts that give optimal investment incentives in this case and, in particular, whether the first best can be achieved using simple real-world contracts, which do not rely on complex revelation mechanisms.

Appendix A

Proofs of Propositions 1, 3, and 4 and proofs of Lemmas 1 and 2 follow.

Proof of Proposition 1. We first show that the seller does not make a renegotiation offer in equilibrium. Given our assumption that an agent makes a renegotiation offer only if it strictly increases its payoff, it suffices to show that the seller has nothing to lose from dropping any renegotiation offer he might make. To see this, consider a subgame starting after renegotiation offers $(\tilde{\rho}_i, \tilde{\rho}')$, $i \in \{B, S\}$, have been made and let $p_T = \max\{p_B, p_S\}$. We argue that the seller's continuation payoff cannot exceed $\max_i (p_T - dc)$. Suppose the seller chooses $d = 0$. The buyer can ensure that his payment does not exceed $p_T$ by withholding any contract the seller may have sent. Hence, the seller's expected continuation payoff (after he has chosen $d = 0$), which is uniquely determined because the contract submission game is zero sum, must be smaller than $p_T$. If the seller chooses $d = 1$, an equivalent argument implies that the seller's expected continuation payoff cannot exceed $p_T - c$. We should note that the seller can ensure the payoff $\max_i (p_T - dc)$ by not making a renegotiation offer, choosing the appropriate delivery decision, and submitting the contract that specifies the price $p_T$ to the court. Because the buyer has only the initial contract to submit, the payment $p_T$ will then be enforced by the court.

Consider now a subgame that results after the buyer (or no agent) has made a renegotiation offer. In a subgame-perfect equilibrium, the seller must choose $d$ to maximize $\max_i (\tilde{\rho}_i, p_S) - d \cdot c$ and then submit the
most profitable contract to the court. If the buyer does not make a renegotiation offer, the seller will thus choose not to deliver if strict inequality holds in case (i) and will choose to deliver in case (ii). For each of these cases, two subcases have to be distinguished:

(i) \( p_1 - p_0 < c(\omega^*, \sigma) \) and \( v(\omega^*, \beta) \leq c(\omega^*, \sigma) \). In this case, \( q = 0 \) is an efficient outcome, and the seller does not want to trade given the initial prices. It is easy to see that there does not exist a renegotiation offer the buyer could make that would increase his payoff. Hence, the buyer does not make a renegotiation offer. In any subgame-perfect equilibrium, the seller will thus not deliver, implying \( q = 0 \) and a transfer payment \( p = p_0 \).

(ii) \( p_1 - p_0 > c(\omega^*, \sigma) \) and \( v(\omega^*, \beta) > c(\omega^*, \sigma) \). In this case, trade would be efficient, but if the buyer does not make a renegotiation offer, the seller is not going to trade because not delivering gives him \( p_0 > p_1 - c \). Because the seller's delivery decision depends only on the difference between the trade and the no-trade price, the buyer will not offer to raise the no-trade price in a subgame-perfect equilibrium. The buyer could send a renegotiation offer to the seller, raising the trade payment to \( p_1^* = p_0 + c + \epsilon \) and leaving the no-trade price unchanged. For all \( \epsilon > 0 \), the seller will respond by delivering and enforcing the payment \( p_1^* \). If the seller also responds by delivering for \( \epsilon = 0 \), then it is optimal for the buyer to offer a new contract with \( p_1^* = p_0 + c(\omega^*, \sigma) \). Indeed, no best response would exist for the buyer if the seller were not to deliver given this offer. Thus, it follows that, in a subgame-perfect equilibrium, the buyer offers a new contract with \( p_1^* = p_0 + c(\omega^*, \sigma) \) and the seller chooses to deliver and then enforces the transfer \( p_1^* \). Note that, because \( v > 0 \), every best response of the buyer specifies that he accepts delivery. Hence, \( q = 1 \).

(iiia) \( p_1 - p_0 > c(\omega^*, \sigma) \) and \( v(\omega^*, \beta) = c(\omega^*, \sigma) \). No trade would be efficient, but if the buyer does not make a renegotiation offer, the seller will choose to deliver. As in case (ii), the buyer can strictly increase his utility by making a renegotiation offer, in this case, one that raises the no-trade payment to \( p_0^* = p_1 - c + \epsilon \). For all \( \epsilon > 0 \), the seller will respond to such an offer by not delivering, implying \( q = 0 \), and enforcing the price \( p_0^* \). Hence, in equilibrium, the buyer will offer \( p_0^* = p_1 - c \) and the seller will not deliver and enforce \( p_0^* \).

(iib) \( p_1 - p_0 > c(\omega^*, \sigma) \) and \( v(\omega^*, \beta) > c(\omega^*, \sigma) \). In this case, the seller wants to deliver given the old prices and trade is efficient. As in case (ia), there is no renegotiation offer the buyer could make that would improve his utility. Thus, the seller will deliver and the transfer is \( p_1 \).

Finally, we need to consider the case \( p_1 - p_0 = c(\omega^*, \sigma) \). Then the seller is indifferent whether to deliver under the terms of the initial contract. If \( v = c \), any decision by the seller results in efficient trade and the buyer is indifferent between receiving the good and paying \( p_1 \) and not receiving it and paying \( p_0 \), so that he has nothing to gain from making a renegotiation offer. If \( v \neq c \), a best response for the buyer only exists if the seller takes the efficient delivery decision, either without receiving a renegotiation offer or after receiving a renegotiation offer that specifies exactly the same prices as the initial contract. \( Q.E.D. \)

Proof of Lemma 1. Let \( k = 0 \). Because production costs are nonnegative, the seller's problem is then to maximize

\[
U^s(p_0, 0, \sigma) = p_0 - h^s(\sigma). \tag{A1}
\]

Because \( h^s \) is increasing in \( \sigma \), the only solution to this problem is \( \sigma = 0 \). Hence, because \( \sigma^* > 0 \), the first claim follows.

Let \( k = k \). Given \( (p_0, k) \), the seller's problem is to maximize

\[
U^s(p_0, k, \sigma) = \int_0^1 (k - c(\omega^*, \sigma))f^s(\omega^*)d\omega^* + p_0 - h^s(\sigma). \tag{A2}
\]

This problem has the same set of maximizers as the problem

\[
\max_{\sigma} \int_0^1 (v(\omega^*, \beta^*) - c(\omega^*, \sigma))f(\omega)d\omega^* d\omega^* - h^s(\sigma) - h^s(\beta^*), \tag{A3}
\]

which, in turn, is the same as

\[
\max_{\sigma} W(\beta^*, \sigma) - \int_0^1 \int_0^1 [c(\omega^*, \sigma) - v(\omega^*, \beta^*)]^+ f(\omega)d\omega^* d\omega^*. \tag{A4}
\]

Because \( c(\omega^*, \sigma) \) is nonincreasing in \( \sigma \) for all \( \omega^* \), the integral added to \( W(\beta^*, \sigma) \) is nondecreasing in \( \sigma \). Hence, this problem cannot have a maximizer \( \sigma < \sigma^* \). \( Q.E.D. \)
Proof of Proposition 3. From Lemma 1 \( \sigma \in \Sigma(k) \Rightarrow \sigma \geq \sigma^* \). Consider (A4) in the proof of Lemma 1. Under the stated condition

\[
\int_0^1 \left\{ c(\omega^*, \sigma) - v(\omega^*, \beta^*) \right\} f(\omega)d\omega = 0
\]

for all \( \sigma \geq \sigma^* \). Thus, for all \( \sigma \geq \sigma^* \) the seller’s maximization problem is equivalent to maximizing \( W(\beta^*, \sigma) \) plus a constant. Consequently, \( \sigma^* \) is the unique optimal investment choice for the seller. Hence, the buyer will also choose \( \beta = \beta^* \). Q.E.D.

Proof of Proposition 4. It follows from Proposition 1 that, if the buyer selects an option contract that makes it optimal for the seller to take the efficient delivery decision, then this contract will not be renegotiated and the seller will indeed trade efficiently. Hence, for all such choices, the buyer’s continuation payoff is as specified in Hermalin and Katz (1993). We must still show that the buyer cannot strictly gain by selecting a contract that would be renegotiated instead of choosing \( k = c(\omega^*, \sigma) \). There are two cases to consider.

First, suppose \( v(\omega^*, \beta) > c(\omega^*, \sigma) \), that is, trade is efficient. It follows from Proposition 1 that renegotiation will occur if and only if the buyer selects \( k < c(\omega^*, \sigma) \). The resulting option contract \( (p_0(k), k) \) will be renegotiated to the contract \( (p_0(k), c(\omega^*, \sigma)) \) and the seller will deliver, so the resulting payment by the buyer is given by \( p_0(k) + c(\omega^*, \sigma) \). If the buyer chooses \( k = c(\omega^*, \sigma) \) instead, the resulting option contract will not be renegotiated, the seller will deliver, and the buyer’s payment is given by \( p_0(c(\omega^*, \sigma)) + c(\omega^*, \sigma) \). Because \( k < c(\omega^*, \sigma) \) implies \( p_0(k) \geq p_0(c(\omega^*, \sigma)) \), it follows that choosing \( k < c(\omega^*, \sigma) \) cannot increase the buyer’s payoff.

Second, suppose \( v(\omega^*, \beta) < c(\omega^*, \sigma) \), that is, trade is inefficient. It follows from Proposition 1 that renegotiation will occur if and only if the buyer selects an option price \( k > c(\omega^*, \sigma) \). The resulting option contract \( (p_0(k), k) \) will be renegotiated to the contract \( (p_0(k) + k - c(\omega^*, \sigma), c(\omega^*, \sigma)) \), the seller will not deliver, and the buyer’s payment is given by \( p_0(c(\omega^*, \sigma)) \). Because \( k > c(\omega^*, \sigma) \) implies \( p_0(c(\omega^*, \sigma)) - p_0(k) \leq k - c(\omega^*, \sigma) \), it follows that the buyer cannot increase his payoff by choosing \( k > c(\omega^*, \sigma) \). Q.E.D.

Proof of Lemma 2. Note that, for all \( q, k \in Q \) and \( k \geq \hat{k}(q) \),

\[
\Sigma(q, k) = \arg\max_{\sigma} -\int_0^1 c(q, \omega^*, \sigma)f(\omega)d\omega - h(\sigma).
\] (A6)

Suppose \( q, k \) satisfies (18). Then (A6) implies \( \max \Sigma(q, k) \geq \sigma^* \).

Suppose \( q, k \) does not satisfy (18). We show that \( \forall k: \max \Sigma(q, k) < \sigma^* \). For \( k \geq \hat{k}(q) \), this follows from (A6). Consider \( k < \hat{k} \). Then

\[
U^k(\sigma, \beta, p_0, \hat{k}(q), q, i) - U^k(\sigma, \beta, p_0, k, q, i) = \int_0^1 \left[ \hat{k}(q) - \max\{k, c(q, \omega^*, \sigma)\} \right] f(\omega)d\omega.
\]

Because production costs are nonincreasing in \( \sigma \), this expression is nondecreasing in \( \sigma \). It follows that \( \forall k < \hat{k}(q) \): \( q \in \Sigma(q, k) \Rightarrow \sigma \leq \max \Sigma(q, k) < \sigma^* \). Hence, if there is no \( q, k \) satisfying (18), then \( \forall q, k: \max \Sigma(q, k) < \sigma^* \). Q.E.D.

Appendix B

- Conditions on the cost functions \( h'(\cdot) \) and \( c(\cdot, \cdot) \) follow.

Assume that \( c \) and \( h' \) are twice continuously differentiable with \( \partial c/\partial \sigma < 0, \partial^2 c/\partial \sigma^2 \geq 0, \partial h'/\partial \omega > 0 \), and \( \partial h'/\partial \sigma > 0 \). For every option price \( k \) and investment level \( \sigma \), there exists a unique \( \omega^*(k, \sigma) \in [0, 1] \) such that

\[
\omega^* < \omega^*(k, \sigma) \Rightarrow c(\omega^*, \sigma) < k
\]

\[
\omega^* > \omega^*(k, \sigma) \Rightarrow c(\omega^*, \sigma) > k.
\]

The seller’s utility function can thus be written as

\[
U^k(p_0, k, \sigma) = \int_0^\omega^*(k, \sigma) \left[ k - c(\omega^*, \sigma) \right] f(\omega)d\omega + p_0 - h'(\sigma).
\] (B1)

Taking the derivative with respect to \( \sigma \), we obtain
For a given $k$, $\omega(k, \sigma)$ is almost everywhere differentiable in $\sigma$ with

$$
\frac{\partial \omega^2(k, \sigma)}{\partial \sigma} = \begin{cases} 
0 & \text{if } k < c(1, \sigma) \\
\frac{\partial c(\omega^2(k, \sigma), \sigma)}{\partial \sigma} & \text{if } c(0, \sigma) < k < c(1, \sigma), \\
\frac{\partial \omega^2}{\partial \sigma} & \text{if } k > c(1, \sigma).
\end{cases}
$$

$\frac{\partial U^2}{\partial \sigma}$ is also almost everywhere differentiable in $\sigma$ with

$$
\frac{\partial^2 U^2}{\partial \sigma^2} = -\int_0^{\omega^2(k, \sigma)} \frac{\partial^2 c(\omega^2, \sigma)}{\partial \sigma^2} f\left(\omega^2\right) d\omega^2 - \frac{\partial^2 h^2(\sigma)}{\partial \sigma^2} - \frac{\partial \omega^2(k, \sigma)}{\partial \sigma} \frac{\partial c(\omega^2(k, \sigma), \sigma)}{\partial \sigma} \cdot f\left(\omega^2(k, \sigma)\right).
$$

Whereas the first two terms in this expression are negative if both $h^2$ and $c$ are convex in $\sigma$, the third term will be strictly positive whenever $c(0, \sigma) < k < c(1, \sigma)$. Hence, assuming that either $h^2$ or $c$ is strictly convex will not suffice to imply that the seller’s maximization problem is strictly concave in $\sigma$ for all $k$. However, it is easy to state a condition that ensures that $h^2$ is sufficiently convex to make the seller’s problem strictly concave. In particular, let

$$
\xi(\sigma) = \max_{\omega^2} \left(\frac{\partial c(\omega^2, \sigma)}{\partial \omega^2}\right)^2 f\left(\omega^2\right)
$$

and suppose that

$$
\frac{\partial^2 h^2(\sigma)}{\partial \sigma^2} > \xi(\sigma).
$$

Then the first derivative $\frac{\partial U^2}{\partial \sigma}$ is strictly decreasing in $\sigma$ for all $k$.

References


