Complementary Patents and Market Structure*

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ABSTRACT: Many high technology goods are based on standards that require several essential patents owned by different IP holders. This gives rise to a complements and a double mark-up problem. We compare the welfare effects of two different business strategies dealing with these problems. Vertical integration of an IP holder and a downstream producer solves the double mark-up problem between these firms. Nevertheless, it may raise royalty rates and reduce output as compared to non-integration. Horizontal integration of IP holders (patent pool, pass through) solves the complements problem but not the double mark-up problem. Vertical integration discourages entry and reduces innovation incentives, while a horizontally integrated firm always benefits from entry and innovation.

Keywords: IP rights, complementary patents, standards, licensing, patent pool, vertical integration.

JEL Classification Codes: L1, L4.

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1. Introduction

High technology products such as cell phones or electronic memory devices are often based on technological standards that require the use of dozens or even hundreds of essential patents owned by different IP holders. By definition an essential patent is strictly necessary for the standard, either because it is legally impossible or prohibitively expensive to do without it.[1] If a downstream firm wants to produce goods that are based on the standard it requires access to each of the essential patents. This gives rise to a *complements effect*: each patent holder does not internalize the negative external effect on the revenues of the other patent holders when setting his royalties, so the sum of all royalties will be inefficiently high. In addition, there is the well known vertical *double mark-up effect* that further raises royalties. Many standard setting organisations see these problems and require their members to charge "reasonable and non-discriminatory" (RAND) royalties.[2] There is a consensus that RAND commitments prevent outright refusal to license and exclusive licensing, but there seem to be no additional constraints implied by RAND, in particular concerning royalties.[3] Thus, a reference to RAND hardly restricts the pricing policies of patent holders.

Firms are using two different business strategies to deal with these externalities. One strategy is vertical integration. For example, in the mobile phone industry some firms such as Nokia or Sony Ericson not only own essential patents to the WCDMA standard, they also produce handsets on the downstream market. By vertically integrating they eliminate the double mark-up problem within the integrated firm (but not between firms). Another strategy is horizontal integration on the upstream market. This can be achieved by forming a patent pool that licences some (or all) essential patents as a bundle at a jointly set royalty rate.[4] Alternatively, one patent holder can acquire additional essential patents or pass through rights from other patent holders and then licence his patent in conjunction with the pass through right to use the other essential patents as well.

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[1] A patent is "legally essential" for a standard if the standard cannot be implemented without infringing the patent. It is "commercially essential" if it is prohibitively expensive to implement the standard without the patent, even if this is technologically feasible. See Layne-Farrar and Lerner (2011). In reality it is not always obvious whether a patent is essential or not. Patent holders have a strong incentive to overstate the importance of their IP rights. Furthermore, it is often unclear whether a patent will survive if it is challenged in court. For a more detailed discussion of these problems see Lemley and Shapiro (2007). In this paper we do not consider these problems and assume that it is common knowledge which patents are in fact essential.

[2] In Europe, most SSOs require royalties to be "fair" in addition.

[3] As Swanson and Baumol (2005) put it: „*It is widely acknowledged that, in fact, there are no generally agreed tests to determine whether a particular license does or does not satisfy a RAND commitment*".

[4] For example, there are two mutually exclusive patent pools on the DVD standard, both of which need to be licensed to produce goods that are compliant with the standard. See Layne-Farrar and Lerner (2011, p. 7).

These different business strategies have been treated differently by anti-trust authorities. Vertical integration is generally considered to be beneficial because it is seen as a remedy against the double mark-up problem. Horizontal integration is more controversial. For many decades patent pools were perceived as a collusion device. While this perception has changed after the U.S. Department of Justice approved the MPEG-2 patent pool in 1997 and the two DVD patent pools shortly thereafter,[5] patent holders that acquire other patents without practicing them are seen with increasing suspicion. Non-practicing patent-holders have been denied injunctive relief and lost profit damages when their patents got infringed on the grounds that they did not use their patents to manufacture themselves.[6]

In this paper we compare the two business strategies of vertical and horizontal integration.[7] We analyze the effects of different market structures on upstream royalties, downstream prices, entry decisions and incentives to innovate. Our model of the downstream market is very general and allows for all kinds of downstream market interaction (competition in prices, quantities, product differentiation, advertising, etc.) as long as a weak regularity condition is satisfied. As a baseline we consider a market structure in which upstream and downstream firms are non-integrated and where linear royalties have to be used upstream. Then we ask how the market outcome changes if some (or all) upstream firms vertically integrate with some downstream firms. Even though vertical integration partially solves the vertical double mark-up problem it may result in higher royalties and less production on the downstream market than non-integration. This is due to the fact that a vertically integrated firm has an incentive to raise its royalty rate in order to raise its rivals' cost. In contrast, horizontal integration of upstream firms is always beneficial and unambiguously reduces the complements problem.

We also consider the use of two-part tariffs. It is well known that two-part tariffs can be used to solve the double mark-up problem in a vertical relationship of two firms that both have market power. We show that it can also be used to solve the complements problem (together with the double mark-up problem) under all market structures. This is particularly simple if all upstream firms are horizontally integrated. If firms are non-integrated there exists a symmetric pure strategy equilibrium in which all upstream firms charge two-part tariffs that

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[5] The Department of Justice and the Federal Trade Commission have softened this stance in their joint report on antitrust and IP issued April 2007 (http://www.ftc.gov/opa/2007/04/ipreport.shtm). Now they acknowledge that including substitute patents need not be anti-competitive. Patent pools will be reviewed according to the rule of reason in the future. See Layne-Farrar and Lerner (2011) and Gilbert (2004) for more on the history of patent pools and the shift of US policy.

[6] See Layne-Farrar and Schmidt (2011).

[7] In our model the optimal solution is always full integration. However, in reality this is rarely achieved because there are too many different parties involved that often have an incentive to free ride on the integration efforts of the other market participants. See Aoki and Nagaoka (2004) and Layne-Farrar and Lerner (2011).

solve the complements and the double mark-up problem. If firms are vertically integrated, however, this equilibrium fails to exist if there are sufficiently many firms. In this case efficiency can be achieved only in asymmetric equilibria.

Perhaps even more important than the effects of market structure on prices are the effects on entry and innovation. We show that vertically integrated firms have an incentive to discriminate against entrants on the downstream market in order to raise their rival’s cost which is not the case for a horizontally integrated or a non-integrated upstream firm. Then we consider the incentives of an upstream firm to innovate and invest in an improvement of the standard. This improvement may reduce downstream production costs, it may make the products based on the standard more valuable to consumers, or it may open the door to new applications. No matter what the benefits of the innovation are, the incentives to innovate are smaller the more firms there are on the upstream market. The reason is that the innovator requires access to all the other patents in the standard. The more IP holders there are, the smaller are the profits that can be generated with any given innovation and the more reluctant the incumbent IP holders are to include an additional essential patent in the standard. Thus, horizontal integration on the upstream market is beneficial because it stimulates innovation.

We conclude that the current shift in US competition policy to permit patent pools for complementary patents is a move in the right direction. However, there is no reason to discriminate against other forms of horizontal integration such as the acquisition of patent rights by non-practicing patent holders. Our analysis shows that vertically integrated and non-integrated patent holders have indeed different incentives to set their royalties, but the presumption that a non-practicing patent holder will always choose higher royalties than a vertically integrated one is not justified.

The remainder of the paper is organized as follows. The next section discusses the related literature. In Section 3 we set up a very general model of a vertically structured industry in which all upstream goods are perfect complements. Section 4 restricts attention to linear royalties and compares a market structure where all firms are non-integrated to market structures where some firms are vertically or horizontally integrated. Section 5 allows for two-part tariffs. In Section 6 we discuss the effect of different market structures on entry on the same or another (unrelated) downstream market, and the incentives of upstream firms to innovate. Section 7 concludes and discusses the application of the model to other industries with complementary inputs such as rail or electricity networks. Most formal proofs are relegated to the Appendix.

2. Related Literature

Our paper is closely related to the growing literature on standard setting, patent pools and complementary patents. Shapiro (2001) discusses the importance of the complements problem for high technology goods that are based on technological standards. He shows that patent pools increase social welfare when all patents are perfect complements and decreases welfare when all patents are perfect substitutes. Lerner and Tirole (2004) show that whether patents are complements or substitutes is endogenous and depends in general on the licensing fees charged for them. They show that welfare-decreasing pools are unstable if independent licensing by pool members outside the pool is possible, while welfare-increasing pools are unaffected. Thus, requiring patent pools to grant permission to independent licensing is a simple safeguard against welfare-decreasing pools. Lerner, Strojwas and Tirole (2007) find that patent pools are indeed more likely to have independent licensing when patents are complements. None of these papers considers the effects of vertical integration nor do they analyse the effects on entry and innovation.

Several recent papers study the formation of a technology standard. Simcoe (2012) and Farrell and Simcoe (2012) analyse how strategic considerations can significantly delay the adoption of a standard. Ganglmair and Tarantino (2011) discuss the incentives to contribute to the development and improvement of a standard. Tarantino (2010) studies the inefficiencies of technology adaption if there are vertically integrated and non-integrated technology providers. Our paper complements this literature by offering a detailed analysis of how the royalty income is determined depending on the underlying market structure, and how this impacts on future entry and innovation decisions.

Layne-Farrar, Padilla and Schmalensee (2007) discuss potential methods for assessing whether licensing terms are “fair, reasonable and non-discriminatory” (FRAND). They argue that patents that make a greater contribution to the value of the standard should be allowed to charge higher royalties. Gilbert and Katz (2011) analyze different sharing rules in patent pools and their impact on the incentives to develop new technology. In our model, all upstream firms are symmetric, so the sharing rule is trivially the equal split.

Our paper is also related to the literature on raising rivals’ costs strategies by vertically integrated firms. Salop and Sheffman (1983, 1987) consider a dominant firm that can affect marginal and average costs of a competitive fringe. They show that the dominant firm will raise its rivals’ cost in order to either foreclose the market or to induce competitors to raise their prices and to relax competition. Ordover, Saloner and Salop (1990) consider a two-stage duopoly model with price competition and differentiated products. In their model there is a foreclosure effect only if downstream firms compete in prices. Goods produced upstream are perfect substitutes. In our model upstream goods are perfect complements and our results hold for any form of downstream competition. Kim (2004) first described the “raising one’s rivals’ cost effect” in a model with complementary upstream goods. However, he only considers Cournot competition with linear demand on the downstream market, while our model allows for a very general model of downstream interaction, and he does not consider two-part tariffs and the implications for entry and innovation.

There are a few recent papers that endogenize the market structure. Reisinger and Tarantino (2012) analyse the incentives for vertical integration in an industry using two complementary input goods. They show that if one of the upstream firms vertically integrates with the downstream firm(s), the other upstream firm will capture some of the additional profits. This effect may render vertical integration unprofitable. Rey and Salant (2012) consider a model in which the number of downstream firms and the number of varieties offered to consumers is determined endogenously. They show that if the number of firms is too high from a social welfare point of view, royalty stacking may improve welfare because it limits entry.

3. The Model

Consider an industry with an upstream and a downstream market. The crucial feature of the upstream market is that the goods offered upstream are perfect complements, all of which are required for downstream production. This is the case in many high technology industries with direct or indirect network externalities in which firms agreed to technological standards to make sure that their products can interoperate or that they are compatible to complementary products. For example, the GSM or WCDMA standards on the telecommunication market guarantee that different handsets can communicate with each other, and the BlueRay and HD-DVD standards ensure that high definition video discs are compatible with DVD players produced by different companies. Typically, a standard requires access to a number of complementary patents that are often owned by different IP holders.

On the upstream market there are m firms, indexed by $u = 1,...,m$. For simplicity we assume that each upstream firm owns one essential patent. The costs for developing patents are sunk. Upstream firms license their patents at non-discriminatory, linear royalties r_u.[8]

On the downstream market there are n symmetric firms, indexed by $d = 1,...,n$. The focus of the analysis is on the royalties charged on the upstream market. Therefore, we want to keep the downstream market as general as possible. We assume that each downstream firm chooses an action vector $x_d \in X_d \subseteq \mathbb{R}^K$ that affects the quantities of the (possibly differentiated) goods that the firm itself and its competitors sell on the downstream market.[9] In the simplest interpretation each firm chooses its quantity q_d directly, so $x_d = q_d$ and $X_d = \mathbb{R}_0^+$. However, downstream firms may also decide on price, advertising, marketing, product differentiation or other business strategies. In this case x_d is a vector with $x_d \in X_d$ where X_d is a subset of some multi-dimensional Euclidean space.

The production of one unit of each of the downstream goods requires full access to all patents. Thus, if downstream firms produce q_d, *d=1,...,n*, then each upstream firm sells $Q = \sum_{d=1}^{n} q_d$ licenses. In the basic model we assume that all fixed costs are sunk and that downstream firms are symmetric and all incur the same marginal production cost k. In addition, each downstream firm has to pay linear royalties $r = \sum_{u=1}^{m} r_u$ for each unit of production. Thus, total marginal cost is $c_d = k + \sum_{u=1}^{m} r_u$.

The time structure of the game is as follows:

1. At stage 1 all upstream firms set their royalty rates simultaneously.
2. At stage 2 all downstream firms observe the royalty rates and choose $x_d \in X_d$ simultaneously. The action profile $x = (x_1,...,x_n)$ determines the quantity q_d sold by each downstream firm and thereby the number of licenses $Q = \sum_{d=1}^{n} q_d$ sold by all upstream firms.

[8] In almost all network industries linear royalties are frequently used. See Section 4 for a more detailed discussion. In Section 5 we also consider the case where upstream firms can charge two-part tariffs and show that our main qualitative results are unaffected.

[9] See Höffler and Kranz (2011) who propose a similar model.

Our main interest is in how royalties are determined at stage 1 under different ownership structures. In order to do comparative statics there has to be a unique equilibrium on the downstream market that determines the total quantity sold downstream and thereby the total demand for licenses upstream. Several authors looked at specific markets and offered sets of sufficient conditions that guarantee existence and uniqueness of a Nash equilibrium.[10] Rather than restricting ourselves to one of these markets we want to keep our model as general as possible and impose the following regularity assumption.

Assumption 1: ***(a)*** *For any vector of royalties* $\vec{r} = (r_1, ..., r_m)$ *and any corresponding vector of marginal costs* $\vec{c} = (c_1, ..., c_n)$ *there exists a unique pure strategy Nash equilibrium* $x^*(\vec{r})$ *of the downstream market game at stage 2 that gives rise to quantities* $\left(q_1\left(x^*(\vec{r})\right), ..., q_n\left(x^*(\vec{r})\right)\right)$ *with* $Q\left(x^*(\vec{r})\right) = \sum_{d=1}^{n} q_d\left(x^*(\vec{r})\right)$.

(b) *If firm d's marginal cost* c_d *increases, its equilibrium quantity* q_d *decreases. Total equilibrium quantity* Q *is continuous and decreasing in the marginal cost* c_d *of each firm* d *,* $d \in \{1, ..., n\}$.

Part (a) of Assumption 1 simply requires the existence of a unique equilibrium in the downstream market game. If there are multiple equilibria a comparative static analysis is possible only with respect to the set of equilibria. Some of our results continue to hold in this case, but the analysis is messy and not insightful. Therefore, we restrict attention to the case of a unique downstream equilibrium. Assumption 1(b) says that the equilibrium production level of each firm is a decreasing function of its own marginal cost and that total output decreases as well. This assumption is very natural and holds quite generally. Dixit (1986) shows that it is satisfied in duopoly models of price and quantity competition with very general demand functions, and in oligopoly models with homogenous goods for both Bertrand and Cournot competition.

Because we do not impose a specific model of downstream competition and do not postulate any specific demand functions derived from the preferences of rational consumers we cannot compute consumer surplus and social welfare directly. This is the price of the generality of our model. However, we can compare the market outcome under different market structures to the outcome that would obtain if there were no contracting problems and

[10] See e.g. Novshek (1985) and Amir (1996).

firms could solve the complements and the double mark-up problem perfectly, i.e. if all upstream and all downstream firms could agree on a set of royalties that maximize total industry profits. We call this the “full integration outcome”. It turns out that in all the cases we consider the market outcome involves higher royalties and lower total quantities than this full integration benchmark. Thus, lower royalties and a higher total quantity increases industry profits. Furthermore, all standard models of oligopoly imply that an increase of Q is associated with an increase of consumer surplus. Therefore we will say that an increase of Q that gets Q closer to the full integration outcome increases social welfare.

For notational simplicity we suppress the reference to the action profile x^* and use $q_d\left(\vec{r}\right)=q_d\left(x^*\left(\vec{r}\right)\right)$ and $Q\left(\vec{r}\right)=Q\left(x^*\left(\vec{r}\right)\right)$ in the following.

4 Optimal Linear Royalties under Different Market Structures

In this section we characterize the royalties that obtain under different market structures. Upstream firms are restricted to use non-discriminatory, linear royalties. Linear royalties are predominantly used in practice.[11] We compare the case of non-integration, where all upstream and downstream firms are owned separately and set royalties and downstream prices independently, to two alternative business models:

- Vertical integration, where some (or all) upstream firms are vertically integrated with some downstream firms.
- Horizontal integration on the upstream market, where several or all patents are owned by one firm. An alternative interpretation of horizontal integration is a patent pool that jointly licenses all patents in a bundle, or a pass through business model, where one patent holder acquires pass through rights from other patent holders and then licences his patent in conjunction with the pass through right to use the other essential patents as well.

[11] Layne-Farrar and Lerner (2011, p. 10) report that linear royalties were used by all the patent pools they investigated. Risk sharing is one reason for the popularity of linear royalties. If downstream demand is uncertain, a linear royalty rate shares the risk between upstream and downstream firms, while a fixed fee shifts all the risk to downstream firms. However, this is outside the scope of this model.

4.1 Non-integration

At stage 1 each upstream firm maximizes $\Pi_u = r_u Q(r_u, r_{-u})$. Note that $c_d = k + \sum_{u=1}^{m} r_u$. Thus, by Assumption 1, Q is a continuous function of r_u and depends on $r = \sum_{u=1}^{n} r_u$ only.

Assumption 2: *The marginal revenue of upstream firm u from increasing its royalty r_u decreases if some other upstream firm j increases its royalty rate r_j, i.e.*

$$\frac{\partial^2 \Pi_u}{\partial r_u \partial r_j} = \frac{\partial Q}{\partial r} + r_u \frac{\partial^2 Q}{\partial r^2} < 0 \qquad (1)$$

The assumption is standard in the literature.[12] It implies the existence of a unique pure strategy equilibrium in the royalty setting game (see Proposition 0 in the Appendix). It also has a natural economic interpretation. If firm *j* increases its royalty rate, total quantity on the downstream market is reduced (by Assumption 1). Assumption 2 requires that this makes it less attractive for firm *u* to raise its royalty rate. Assumptions 1 and 2 are maintained for the rest of the paper.

Under non-integration each upstream firm maximizes its profits $\Pi_u^{NI} = r_u Q(r_u, r_{-u})$. Because all firms are symmetric in equilibrium they all charge the same royalty rate r_u^{NI} that is fully characterized by the first order condition

$$\frac{\partial \Pi_u^{NI}}{\partial r_u} = \underbrace{Q}_{>0} + \underbrace{\frac{\partial Q}{\partial r}}_{<0} \cdot \underbrace{r_u^{NI}}_{>0} = 0 \qquad (2)$$

where the superscript *NI* stands for "Non-integration".

As a reference point, suppose that all upstream and all downstream firms can agree on a set of royalties that maximize total industry profit, but they cannot restrict the actions chosen on the downstream market. This is called the "full integration" benchmark. Total industry profit is given by

$$\Pi^{FI} = \underbrace{r_u Q(r_u, r_{-u})}_{\text{upstream profit of firm } u} + \underbrace{\sum_{j \neq u} r_j Q(r_u, r_{-u})}_{\text{profits of all other upstream firms } j \neq u} + \underbrace{\sum_{d=1}^{n} \Pi_d(r_u, r_{-u})}_{\text{downstream profits}} \qquad (3)$$

[12] The same assumption is imposed in any Cournot game to guarantee existence of a pure strategy equilibrium. See Novshek (1985).

Comparing total profits under full integration (3) to the objective function of a single upstream firm, we see that each upstream firm does not take into account the impact of its own royalty rate on the profits of all other upstream firms, nor on the profits of all downstream firms nor on consumer surplus.

Proposition 1 [Non-Integration]: *Under non-integration equilibrium royalties are strictly greater and total quantity is strictly smaller than in the full integration benchmark.*

Proof: See Appendix.

The intuition for Proposition 1 is that by increasing its royalty rate each upstream firm u exerts two negative externalities:

- it reduces total quantity Q which reduces the profits of the other upstream firms (complements effect),
- it raises the total royalty burden which reduces the profits of the downstream firms (double mark-up effect).

Because each upstream firm does not take these negative externalities into account, royalties are inefficiently high.

The complements effect has first been observed by Cournot (1838, Chapter 9). It stems from the fact that the goods produced by the upstream firms are perfect complements that are sold by independent firms. The double mark-up effect is due to the vertical chain of producers that all have market power. Upstream firms have a monopoly on their patents that are essential inputs for downstream firms that also have market power and impose an additional mark-up when they sell to consumers.

4.2 Vertical Integration

Suppose now that *l* upstream firms and *l* downstream firms vertically integrate, $l \leq m, n$, one upstream firm with one downstream firm each. Thus, we now have *l* vertically integrated firms, *m-l* non-integrated upstream firms, and *n-l* non-integrated downstream firms. At the first stage the non-integrated upstream firms and the upstream divisions of the vertically integrated firms set linear royalties r_i, $i = 1,...,m$. At the second stage the non-integrated downstream firms and the downstream divisions of the vertically integrated firms choose action vectors x_d giving rise to quantities q_d, $d = 1,...,n$, on the downstream market.

Note that if firms would charge the same royalties under non-integration and under vertical integration, then the total quantity produced would be larger the more upstream firms are vertically integrated. This follows immediately from the fact that a vertically integrated firm does not have to pay royalties to itself. Therefore the marginal cost of each vertically integrated firm would be lower than the cost of a non-integrated firm. By Assumption 1, total equilibrium quantity Q increases if the marginal cost of one firm decreases. Therefore, Q would increase if upstream firms vertically integrate, still assuming that royalties remain unchanged. This suggests that vertical integration is beneficial because it raises total quantity. But, of course, if the market structure changes firms have different incentives to set royalties.

What royalties will be chosen under vertical integration? When a vertically integrated firm sets its royalty rate, it internalizes the effect on the profits of its own downstream division. Thus, vertical integration solves the double mark-up problem within each firm. However, there are still three negative externalities:

1. The double mark-up problem across firms remains, because firm i does not take into account the effect of its own royalty r_i on firm j's downstream profit.
2. Furthermore, a vertically integrated firm does not internalize the effect of its royalty rate on the upstream profits of the other firms. Thus, vertical integration does not solve the complements problem.
3. Finally, there is a new externality that does not exist under non-integration nor under horizontal integration. This is a "raising one's rivals' costs" effect: The higher the royalty charged by firm i the higher are the costs of the other firms active on the downstream market, while firm i's costs are unaffected. This induces firm i to raise its royalties in order to raise its rivals' costs.

Proposition 2 [Vertical Integration]:

(a) If some (or all) upstream firms are vertically integrated equilibrium royalties are higher and total quantity is still lower than in the full integration benchmark.

(b) If there are some vertically integrated and some non-integrated upstream firms, then the vertically integrated firms may choose higher or lower royalties than non-integrated upstream firms. Furthermore, total quantity Q (and social welfare) may increase or decrease when an upstream firm vertically integrates with a downstream firm.

Proof: See Appendix.

Part (a) of Proposition says that vertical integration alone is not sufficient to achieve the full integration benchmark. This is not surprising because each vertically integrated firm does not internalize the complements effect. However, one might have expected that vertical integration increases social welfare because a vertically integrated firm internalizes the double-mark up effect. However, part (b) of the proposition tells us that this need not be the case. Vertical integration may lead to higher royalties and a less efficient market outcome.

To see the intuition for this result consider a vertically integrated firm v hat chooses its royalty r_v in order to maximize the sum of profits in its upstream and downstream division:[13]

$$\Pi_v = \underbrace{r_v Q^{VI}\left(\vec{r}\right)}_{upstream\_profit} + \underbrace{q_v\left(\vec{r}\right)\cdot\left[P\left(Q^{VI}\left(\vec{r}\right)\right)-c_v\left(\vec{r}\right)\right]}_{downstream\_profit} \tag{4}$$

Differentiating with respect to r_v the FOC for the optimal r_v requires:

$$\frac{\partial\Pi_v}{\partial r_v} = \underbrace{Q^{VI} + \frac{\partial Q^{VI}}{\partial r_v}\cdot r_v}_{\substack{\text{same as under}\\ \text{non-integration}}} + \underbrace{\left[\frac{\partial P}{\partial Q}\cdot\frac{\partial Q^{VI}}{\partial r_v}-1\right]\cdot q_v^{VI}}_{\substack{<0\\ \text{internalization of}\\ \text{vertical double mark-up}}} + \underbrace{\left[P-k-\sum_{j=1}^{n} r_j\right]\cdot\frac{\partial q_v^{VI}}{\partial r_v}}_{\substack{>0\\ \text{raising one's rivals' costs}}} \tag{5}$$

The first two terms correspond to the FOC under non-integration: An increase in r_v raises revenues per unit of output, but it reduces the quantity of output. The last two terms reflect the effect of an increase of r_v on downstream profits and have no analogue under non-integration.

Consider the third term first: By increasing its royalty rate r_v firm v raises the costs of all downstream firms which increases the market price. However, it also increases the cost of its own downstream division, so profits of the downstream division are reduced. Because firm v internalizes this vertical double mark-up problem it has an incentive to moderate its royalty rate as compared to a non-integrated upstream firm.

However, there is a forth effect that works in the opposite direction: By raising its royalty rate firm v increases the marginal costs of its downstream competitors $i \neq v$. Thus, in

[13] Note that the royalty income from its own downstream division is a cost of the downstream division and thus cancels out in the profit function. However, it will be convenient to keep these two terms separate in order to facilitate the comparison of the first order conditions.

the downstream continuation equilibrium the quantities chosen by all other firms are reduced while the quantity of firm v goes up, so firm v receives the mark-up, $P-c_i$, on a larger quantity. Thus, the forth term gives an additional incentive to raise royalties as compared to a non-integrated upstream firm.[14] This "raising one's rivals' cost effect" implies that each vertically integrated firm has an incentive to raise its royalty rate in order to improve its own market position to the detriment of its rivals. However, there is a prisoners' dilemma. In equilibrium all vertically integrated firms choose the same royalty, nobody has a competitive advantage, and everybody would be better off if all firms could jointly reduce their royalties. This explains why social welfare may be reduced by vertical integration.

4.3 Horizontal Integration

We now consider the possibility that some upstream firms integrate horizontally, i.e. either one patent holder buys up some other essential patents, or patent holders form a patent pool, or one patent holder offers pass through rights. In either case the "horizontally integrated" firm bundles its IP rights and licenses them at a joint royalty rate on the downstream market.

Proposition 3 [Horizontal Integration]: *(a) If some (or all) upstream firms integrate horizontally, total equilibrium royalties are unambiguously reduced and the total quantity sold on the downstream market (and social welfare) unambiguously goes up.*

(b) If all upstream firms are horizontally integrated the complements effect disappears and there is no raising one's rivals' cost effect, but the double mark-up effect remains. Therefore, the royalty charged by the upstream firm is too high and downstream quantity is too low as compared to the full integration benchmark, but the allocation is more efficient than the non-integration outcome.

Proof: See Appendix.

Proposition 3(a) shows that - in contrast to the case of vertical integration - a horizontal merger unambiguously reduces royalties and increases total output and social welfare. Hence, horizontal integration is always more profitable and more efficient than non-integration, but – because of the double mark-up problem – it does not achieve the full integration benchmark.

[14] "Raising one's rivals' cost effects" were first described by Salop and Sheffman (1983, 1987). Kim (2004) shows that there is such an effect in a vertically integrated industry with Cournot competition downstream.

The royalty rate chosen by a horizontally integrated upstream firm is larger than the royalty in the full integration benchmark because of the double mark-up problem. However, the more competition there is on the downstream market, the smaller is the mark-up. In the limit, if downstream competition becomes fully competitive, the double mark-up problem disappears and the royalty chosen by a horizontally integrated upstream firm approaches the royalty rate that maximizes total industry profits.

This result suggests that if the downstream market is sufficiently competitive, horizontal integration outperforms vertical integration. Indeed, in the example of a Cournot model with linear demand three downstream firms are already sufficient to render horizontal integration superior to vertical integration:

Example 1: *Suppose that there is Cournot competition downstream and that the demand function is linear. If there are more than two firms on the downstream market, horizontal integration yields a higher output on the downstream market than vertical integration.*

Proof: see Appendix.

The example suggests that horizontal integration outperforms vertical integration when the downstream market is sufficiently competitive, because more competition downstream makes the double mark-up problem disappear.

5 Two-part Tariffs

So far we assumed that upstream firms are restricted to use linear royalties which is the prevalent case in reality. However, firms could also use two-part tariffs. It is well known that two-part tariffs can be used to solve the double mark-up problem. If all firms are horizontally integrated (e.g. by a patent pool) it is very simple (and a dominant strategy) to implement the full integration outcome: Set the linear royalty such that downstream firms are induced to charge the monopoly price and choose the fixed fee such that it extracts all downstream profits.

In this section we show that if not all firms are horizontally integrated they can still use two-part tariffs to solve the complements problem and to implement the full integration outcome in a subgame perfect equilibrium (SPE). However, in contrast to the case of full horizontal integration this requires coordination among the IP holders. We will show that coordination can be difficult and that it may be more difficult when firms are vertically integrated than when they are non-integrated.

Proposition 4 [Two-part Tariffs]: *Suppose that two-part tariffs can be used.*
(a) Horizontal Integration: *If all upstream firms are horizontally integrated there exists a unique dominant strategy SPE that implements the full integration outcome.*
(b) Non-integration: *If all firms are non-integrated there exists a symmetric pure strategy SPE that implements the full information outcome, but it is not in dominant strategies.*
(c) Vertical Integration: *If all* m *upstream firms are vertically integrated there exist pure strategy SPE that implement the full integration outcome. However, there exists an* $\underline{m}$ *such that if* $m \geq \underline{m}$ *then all such equilibria must be asymmetric.*

Proof: See Appendix.

Thus, two-part tariffs can be used to increase total output, total industry profit and social welfare. If all patent holders are horizontally integrated (e.g. in a patent pool) it is a dominant strategy to charge royalties that implement the full integration benchmark. If all patent holders are non-integrated there exists one symmetric equilibrium that implements the full integration benchmark, but there are also many other symmetric equilibria that are inefficient. For example, it is always an equilibrium that all upstream firms charge fixed and/or linear royalties that are so high that no downstream firm wants to license. If there are sufficiently many vertically integrated firms it is still possible to implement the full integration outcome, but there does not exist an efficient symmetric pure strategy equilibrium. Thus, horizontal integration makes it very easy to coordinate on the full integration outcome while vertical integration makes the implementation problem more difficult.

6 Entry and Innovation

In the previous sections we looked at a static market in which the companies and the products and technologies they sold were exogenously given. In this section we want to discuss some dynamic features of these markets. What are the incentives of potential entrants to enter the downstream market depending on the market structure? How is this affected by vertical and horizontal integration? How does the number of essential patent holders affect the incentives to innovate and to come up with new technologies that complement the existing products?

6.1 Entry on the Downstream Market

Suppose that a potential entrant considers entering the downstream market. The entrant can produce a potentially differentiated good with marginal cost $k_e \geq 0$ and he has to incur an entry cost $K \geq 0$ that is sunk.

Assumption 3: *For any given vector of royalties* $\vec{r} = (r_1, ..., r_m)$ *total output on the downstream market is larger if the entrant enters the market than if he does not enter.*

This assumption excludes cases where entry causes other firms to leave the downstream market which could potentially reduce total output.

The timing is as follows: First, the entrant decides whether to enter the market. Then upstream firms decide on their linear royalties. Finally firms compete on the downstream market and profits are realized.

Proposition 5 [Entry Downstream]: *Suppose Assumption 3 holds. Then a horizontally integrated upstream firm unambiguously benefits from entry on the downstream market. In contrast, a vertically integrated firm may be hurt by entry and may charge prohibitively high royalties in order to foreclose the entrant.*

Proof: See Appendix.

The intuition for Proposition 5 is as follows. By Assumption 3 entry increases total output on the downstream market. Thus, if a horizontally integrated firm keeps its royalty rate unchanged, it will sell more licenses after entry which increases its profits. By adjusting its royalty rate to the new market structure it may increase its profits even further. Thus, a horizontally integrated upstream firm always benefits from entry and wants to encourage it.

For a vertically integrated patent holder, however, there is a trade-off. On the one hand his upstream division benefits from the extension of the downstream market. On the other hand, the increased competition reduces the profits of his downstream division. To mitigate the increased competition the vertically integrated firm may want to raise the entrant's cost by increasing its royalty rate. It may even want to foreclose the entrant by charging a prohibitively high royalty rate for its essential patent.

In the Appendix we offer a simple example of a market with differentiated products and Bertrand competition illustrating these effects. If the products of the vertically integrated firm and of the potential entrant are close substitutes, i.e. if the substitution coefficient is larger than some cut-off value, then the vertically integrated firm finds it optimal to foreclose the market by charging a prohibitively high royalty rate. But if the substitution coefficient is below the cut-off, i.e. if downstream competition is less intense, then the vertically integrated firm will accommodate entry.

6.2 Innovation on the Upstream Market

What are the implications of different market structures on the incentives to innovate and to come up with new technologies on the upstream market? Suppose that a company has an idea for an innovation that improves the quality of the standard. This may be an additional feature that makes it possible to use the technology for new applications, to reduce the cost to employ the technology in downstream production, or to raise the benefits of consumers from using the downstream product. However, the innovation can be used by downstream firms and/or consumers only if it is included in the standard. Thus, an innovator has to get his patent included in the standard before he can license it. This requires approval by all other patent holders who participate in the standard already.

We model upstream innovation as follows: To develop the innovation and to protect it by a patent the innovator has to incur a fixed investment cost $I > 0$. The innovation can be used only if the existing upstream firms include it in the standard. The innovation raises consumers' willingness to pay and/or lowers production costs. This raises the profits that can be made on upstream and downstream markets. By how much upstream profits increase depends on the number of upstream patent holders. The analysis of Section 4 has shown that if linear royalties are used total upstream profits are higher the fewer upstream firms there are. Let $\Pi(m)$ denote the total equilibrium profit without the innovation it there are $m \geq 1$ essential patent holders and let $\Pi(m) + \Delta(m)$ denote the total profit if the innovation is included in the standard. Two cases have to be distinguished:

1. *Inside Innovator:* The potential innovator already owns one of the patents that are essential to the standard. In this case including the innovation in the standard increases total upstream profits from $\Pi(m)$ to $\Pi(m) + \Delta(m)$.

2. *Outside Innovator:* The potential innovator does not yet own a patent that is essential to the standard. In this case including the innovation in the standard increases the number of essential patent holders from m to $m+1$ and yields total upstream profits of $\Pi(m+1)+\Delta(m+1)$.

Assumption 4: *Initially there are $m \geq 1$ symmetric firms each holding one patent that is essential to the standard. A new patent is included in the standard if and only if all existing patent holders agree to it.*

The timing is as follows: First, the innovator decides whether to pursue an innovation costing $I > 0$ and yielding additional upstream profits $\Delta(m)$ that may depend on the number of essential patent holders. Then all existing patent holders have to approve inclusion of the innovation in the standard. In this section we do not model the royalty setting and downstream competition explicitly but assume that the total profit functions $\Pi(m)$ and $\Delta(m)$ are given exogenously and satisfy the results of Section 4.

Proposition 6 [Innovation Upstream]: *Suppose Assumption 4 holds.*

(a) An inside innovator undertakes an innovation if and only if $I \leq \frac{\Delta(m)}{m}$.

(b) *An outside innovator undertakes an innovation if and only if* $I \leq \frac{\Pi(m+1)+\Delta(m+1)}{m+1}$

and $\Delta(m+1) > \underbrace{\frac{\Pi(m)}{m}}_{>0} + \underbrace{\Pi(m)-\Pi(m+1)}_{>0}$.

(c) *Innovation incentives increase as the number of upstream firms decreases and are maximized under horizontal integration.*

<u>Proof of Proposition 6:</u> (a) An innovation by an inside innovator is always approved by all the other essential patent holders. It increases the total profit on the upstream market without changing the number of essential patent holders. The profit of each upstream firm (including the innovator) increases from $\frac{\Pi(m)}{m}$ to $\frac{\Pi(m)+\Delta(m)}{m}$. Thus, the innovation pays off for the inside innovator if and only if $I \leq \frac{\Delta(m)}{m}$.

(b) An outside innovator has to get approval of all existing essential patent holders to include his innovation in the standard. Without the innovation each existing patent holder gets $\frac{\Pi(m)}{m}$. With the innovation he gets $\frac{\Pi(m+1)+\Delta(m+1)}{m+1}$. Thus, each existing patent holder will approve if and only if

$$\frac{\Pi(m+1)+\Delta(m+1)}{m+1} \geq \frac{\Pi(m)}{m} \quad \Leftrightarrow \quad \Delta(m+1) \geq \underbrace{\frac{\Pi(m)}{m}}_{>0} + \underbrace{\Pi(m)-\Pi(m+1)}_{>0} > 0 .$$

Furthermore, the innovation is profitable only if the innovators share in total profits exceeds his investment cost, i.e. if $I < \frac{\Pi(m+1)+\Delta(m+1)}{m+1}$.

(c) By Proposition 3 we know that for all $m \geq 1$ cumulative royalties are higher than the royalties that a fully integrated monopolist would choose. Furthermore, cumulative royalties decrease as the number of patent holders decreases. Thus, the smaller m the lower are cumulative royalties and the higher are upstream profits. Thus, upstream profits and social welfare are maximized under horizontal integration. *Q.E.D.*

A few remarks are noteworthy. First, if there is a horizontally integrated upstream firm that comes up with an idea for an innovation, then it will invest if and only if $I \leq \Delta(1)$. From the perspective of the upstream market this is the efficient outcome. If there is more than one upstream patent holder and/or if the innovator is an outside innovator there will be too little innovation because the innovator has to share the fruits of his innovation with the other essential patent holders. Second, an outside innovator has to compensate the incumbent essential patent holders for the fact that having one additional essential patent holder in the standard reduces total upstream profits and that these reduced profits now have to be shared by $m+1$ rather than by m parties. Thus, an outside innovator will make only large innovations that increase total upstream profits significantly. An inside innovator, on the other hand, does not face this constraint and will engage also in small innovations as long as $I \leq \frac{\Delta(m)}{m}$. This suggests that outside innovators tend to make large innovations while inside innovators are more likely to make small ones. Finally innovation becomes more difficult the more essential patent holders there are, because the fruits of the innovation have to be shared with all of them.

7 Conclusions

If different IP holders own complementary patents that are all essential to a standard, several externalities arise that affect their pricing decisions. In a general model of downstream competition we have shown that horizontal integration has positive effects on total output and tends to increase social welfare, while the effects of vertical integration are ambiguous. Horizontal integration eliminates the complements effect and induces lower prices and higher quantities on the downstream market. Furthermore, a horizontally integrated firm benefits from downstream market entry and encourages innovation upstream. Vertical integration, on the other hand, solves the double-mark-up problem between the two merging firms, but it gives rise to a raising one's rivals' cost effect. The net effect is ambiguous and may increase prices and reduce output and social welfare. Furthermore, vertically integrated firms compete against new market entrants and have an incentive to discriminate against them. Finally, vertical integration does not affect the problem that an innovator needs permission by all upstream IP holders to join the standard, so it does not encourage innovation. These results suggest that the current shift in US and EU competition policy to permit patent pools for complementary patents is beneficial. It also suggests that companies that buy patents in order to bundle them or to pass them through as a bundle to downstream firms perform an important welfare increasing function. On the other hand, our analysis shows that vertical integration can have ambiguous effects and should be seen with more caution, in particular when entry and innovation are of crucial importance for the development of the market.

Our model also applies to industries that require access to a physical network such as electricity, railways or fixed-line telecommunications, if the network consists of separate parts that complement each other. For example, there are often regional monopolies that own separate parts of the electricity grid, of the railway network or of the telecommunications infrastructure. If a downstream firms wants to offer services that are based on the network it often need access to the entire network. In this case the different parts of the network are perfect complements and the results of this paper apply.

Appendix

Proposition 0: *There exists a unique pure strategy equilibrium in the royalty setting game at stage 1.*

Proof of Proposition 0: Assumption 2 implies that the profit function of each upstream firm is globally concave in r_u,

$$\frac{\partial^2 \Pi}{\partial r_u^2} = \frac{\partial Q}{\partial r} + \frac{\partial Q}{\partial r} + r_u \frac{\partial^2 Q}{\partial r^2} < \frac{\partial Q}{\partial r} + r_u \frac{\partial^2 Q}{\partial r^2} < 0$$

Thus, by the well known existence proof for concave games (Debreu, 1952), a pure strategy equilibrium exists. In equilibrium the FOCs

$$\frac{\partial \Pi_u}{\partial r_u} = Q + r_u \frac{\partial Q}{\partial r} = 0 \tag{6}$$

must be satisfied. Suppose that there are two equilibria with corresponding royalty vectors $(r_1^A, ..., r_m^A)$ and $(r_1^B, ..., r_m^B)$. Let $r^A = \sum_{u=1}^{m} r_u^A$ and $r^B = \sum_{u=1}^{m} r_u^B$, and suppose wlog that $r^A > r^B$. This implies that $Q(r^A) < Q(r^B)$. Comparing the FOCs for r_u in the two equilibria we have $r_u^A Q'(r^A) > r_u^B Q'(r^B)$. Summing up over all u this implies $r^A Q'(r^A) \geq r^B Q'(r^B)$. However, Assumption 2 implies $\frac{\partial Q}{\partial r} + r\frac{\partial^2 Q}{\partial r^2} < 0$ which in turn implies

$$\int_{r^B}^{r^A} [Q'(r) + rQ''(r)]dQ = \left[rQ'(r)\right]_{r^B}^{r^A} = r^A Q'(r^A) - r^B Q'(r^B) < 0$$

a contradiction. Thus, $r_A = r_B$. Symmetry implies $r_1^A = ... = r_m^A$ and $r^B = ... = r_m^B$. Thus, equilibrium royalties are unique. *Q.E.D.*

Proof of Proposition 1: Let $r^{FI} = \sum_{u=1}^{n} r_u^{FI}$ denote the cumulative royalty rate under full integration ("*FI*"). The first order conditions for the maximization of total industry profits require for all $u = 1, ..., n$:

$$\frac{\partial \Pi^{FI}}{\partial r_u} = \underbrace{Q(r^{FI})}_{>0} + \underbrace{\frac{\partial Q(r^{FI})}{\partial r} \cdot r_u^{FI}}_{<0} + \underbrace{\sum_{j \neq u} r_j^{FI} \cdot \frac{\partial Q(r^{FI})}{\partial r}}_{\substack{<0 \\ \text{complements effect}}} + \underbrace{\sum_{d=1}^{m} \frac{\partial \Pi_d}{\partial r_u}}_{\substack{<0 \\ \text{double mark-up effect}}} = 0 \tag{7}$$

Because of symmetry we have $r_1^{FI} = ... = r_m^{FI}$. If all upstream firms choose the optimal royalties under full integration total quantity is $Q(r^{FI})$. Comparing (7) to (2) it is straightforward to see that if $r_u^{NI} = r_u^{FI}$ then the first derivative of each firm's profit function is strictly positive. Thus, this cannot be an equilibrium. Each firm would have an incentive to increase its royalty r_u until (2) is satisfied. Hence, $r_u^{NI} > r_u^{FI}$. *Q.E.D.*

Proof of Proposition 2:

(a) A vertically integrated firm v, $v \in \{1,...,l\}$ chooses its royalty rate to maximize $\Pi_v^V = \underbrace{r_v Q^{VI}\left(\vec{r}\right)}_{upstream\_profit} + \underbrace{q_v\left(\vec{r}\right)\cdot\left[P\left(Q^{VI}\left(\vec{r}\right)\right) - c_v\left(\vec{r}\right)\right]}_{downstream\_profit}$, where $Q^{VI}(\vec{r})$ denotes total output if all upstream firms are vertically integrated. The FOC for this maximization problem is

$$\frac{\partial \Pi_v^V}{\partial r_v} = Q^{VI} + \frac{\partial Q^{VI}}{\partial r_v}\cdot r_v + \left[\frac{\partial P}{\partial Q}\cdot\frac{\partial Q^{VI}}{\partial r_v} - 1\right]\cdot q_v^{VI} + \left[P - k - \sum_{j=1}^{n} r_j\right]\cdot\frac{\partial q_v^{VI}}{\partial r_v} = 0 \qquad (8)$$

Total industry profit is given by $\Pi = \Pi_v + \sum_{u\neq v}\Pi_u + \sum_{d\neq v}\Pi_d$. The first order condition for the maximization of total industry profit with respect to r_v is given by

$$\frac{\partial \Pi}{\partial r_v} = \underbrace{\frac{\partial \Pi_v}{\partial r_v} Q(r) + \frac{\partial Q(r)}{\partial r}\cdot r_v + \left[\frac{\partial P}{\partial Q}\cdot\frac{\partial Q}{\partial r_v} - 1\right]\cdot q_v + \left[P - k - \sum_{j=1}^{n} r_j\right]\cdot\frac{\partial q_v}{\partial r_v}}_{\text{Effect on profit of firm v}} + \underbrace{\sum_{j\neq v}\frac{\partial \Pi_d}{\partial r_u}}_{\substack{<0 \\ \text{negative effect} \\ \text{on profits of all} \\ \text{other firms}}} = 0 \qquad (9)$$

Note that total industry profit is maximized if all patent holders choose the full integration royalties r_u^{FI} defined by (7) are chosen. Suppose all patent holders choose these royalties, so total downstream quantity is $Q(r^{FI})$. Then (9) implies

$$Q(r^{FI}) + \frac{\partial Q(r^{FI})}{\partial r_v}\cdot r_v^{FI} + \left[\frac{\partial P}{\partial Q}\cdot\frac{\partial Q(r^{FI})}{\partial r_v} - 1\right]\cdot q_v(r^{FI}) + \left[P - k - \sum_{j=1}^{n} r_j\right]\cdot\frac{\partial q_v(r^{FI})}{\partial r_v} = \underbrace{-\sum_{j\neq v}\frac{\partial \Pi_j}{\partial r_u}}_{>0} \qquad (10)$$

Note that the left hand side of this equation is the derivative of firm v's profit function with respect to r_v at $r_v = r_v^{FI}$. Because this derivative is strictly positive, firm *v* has an incentive to further increase its royalty rate. Thus, $r_v^{VI} > r_u^{FI}$.

(b) We give two examples to show that the effect on Q can go in both directions. Suppose that $m = n$, all firms are symmetric and compete in quantities downstream. If the demand function is linear, it is easy to compute that

$$r_u^{VI} = \frac{(a-k)\cdot(n+3)}{n^2+4n-1} > \frac{a-k}{n+1} = r_u^{NI}.$$

Thus, with linear demand firms charge higher royalties when they are vertically integrated than when they are not.

If the demand function is given by $P = Q^{-\frac{1}{\eta}}$ and if $-\eta > n$ it can be shown that

$$r_i^{VI} = \frac{k\left(n^2\eta - n + 2\eta n - 1 - \eta\right)}{n\left(2n\eta^2 + n - n^2\eta - \eta n - \eta^2\right)} < \frac{k}{\eta - n} = r_i^{NI},$$

so royalties charged by vertically integrated firms are smaller than under non-integration.

If vertical integration yields lower royalties than non-integration it unambiguously increases social welfare. However, if vertical integration yields higher royalties than non-integration the effect is ambiguous. It may still yield a more efficient market outcome because vertically integrated firms are not distorted by the royalties that they pay to themselves. Kim (2004, p. 245) shows that in the case of a Cournot model with linear demand, if the number of vertically integrated firms is not too large then vertical integration yields a total quantity that is smaller than the total quantity produced under non-integration. In this example vertical integration reduces total output, total industry profits and social welfare. *Q.E.D.*

Proof of Proposition 3:

(a) In equilibrium, the first order condition $\frac{\partial \Pi_u}{\partial r_u} = Q + \frac{\partial Q}{\partial r} \cdot r_u = 0$ has to hold for all upstream firms $u = 1,\dots,m$. Consider two upstream markets with m^1 and m^2 firms respectively, $m^1 > m^2$. Summing up over all firms we get

$$m^1 \cdot Q(r^1) + m^1 \cdot r^1 \cdot \frac{\partial Q(r^1)}{\partial r} = 0$$

$$m^2 \cdot Q(r^2) + m^2 \cdot r^2 \cdot \frac{\partial Q(r^2)}{\partial r} = 0$$

where $r^i = \sum_{u=1}^{m} r_u^i$, $i \in \{1,2\}$. Subtracting the second equation from the first we have

$$m^1 \cdot Q(r^1) - m^2 \cdot Q(r^2) + m^1 \cdot r^1 \cdot \frac{\partial Q(r^1)}{\partial r} - m^2 \cdot r^2 \cdot \frac{\partial Q(r^2)}{\partial r} = 0$$

Suppose that $r^2 \geq r^1$. Assumption 2 is equivalent to the assumption that $\frac{\partial Q}{\partial r} + r\frac{\partial^2 Q}{\partial r^2} < 0$ which implies

$$\int_{r^1}^{r^2} [Q'(r) + rQ''(r)]dr = \left[rQ'(r)\right]_{r^1}^{r^2} = r^2 Q'(r^2) - r^1 Q'(r^1) \leq 0$$

Thus, we must have

$$m^1 \cdot Q(r^1) - m^2 \cdot Q(r^2) \leq 0$$

However, because $m^2 < m^1$ this implies $Q(r^2) > Q(r^1)$ which implies $r^2 < r^1$, a contradiction. Thus, we must have $r^1 > r^2$. Note that $r^1 > r^2$ implies that all downstream firms have lower costs with m^2 than with m^1 upstream firms, so downstream prices are lower and the quantity sold on the downstream market is higher if the number of upstream firms decreases.

(b) A horizontally integrated firm chooses its royalty rate to maximize $\Pi^H = r\,Q^{HI}(r)$. The FOC for this maximization problem is

$$\frac{\partial \Pi^H}{\partial r} = Q^{HI} + \frac{\partial Q^{HI}}{\partial r} \cdot r^H \tag{11}$$

The first order condition for the maximization of total industry profits can be rewritten as:

$$\frac{\partial \Pi}{\partial r} = \underbrace{Q(r^{FI}) + \frac{\partial Q(r^{FI})}{\partial r} \cdot r^{FI}}_{\text{same as under non-integration}} + \underbrace{\sum_{d=1}^{n} \frac{\partial \Pi_d}{\partial r}}_{\substack{<0 \\ \text{internalize double margin effect} \\ \text{on downstream firms}}} = 0 \tag{12}$$

Suppose the horizontally integrated upstream firm chooses the optimal royalty rate under full integration r^{FI}, so total downstream quantity is $Q(r^{FI})$. Then we have

as:

$$\frac{\partial \Pi^H}{\partial r} = Q(r^{FI}) + \frac{\partial Q(r^{FI})}{\partial r} \cdot r^{FI} = \frac{\partial \Pi}{\partial r} - \underbrace{\sum_{d=1}^{n} \frac{\partial \Pi_d}{\partial r}}_{<0} > 0 \tag{13}$$

Hence, the horizontally integrated firm will choose a royalty that is larger than r^{FI} and total quantity on the downstream market will be lower. *Q.E.D.*

Example 1: If all upstream firms are horizontally integrated, total quantity is given by $Q^{HI}(n) = \frac{n(a-c)}{2b(n+1)}$. On the other hand, if all *m* upstream firms are vertically integrated they will set royalties that exclude all independent downstream firms from the market. In this case total quantity is given by $Q^{VI} = \frac{2m(a-c)}{b(m(m+3)+(m+1))}$. Note that Q^{HI} is strictly increasing in *n* while Q^{VI} is strictly decreasing in m. For *n=3* total quantity is higher under HI than total quantity under VI for *m=2*.

<u>Proof of Proposition 4:</u> (a) Obvious.

(b) In any equilibrium all downstream profits must be extracted. If this was not the case each upstream firm would have an incentive to further raise the fixed fee of its royalties. Consider now an equilibrium candidate where the linear royalties are sufficiently small that all downstream firms want to produce. Suppose that the sum of all linear royalties is larger (smaller) than the royalty rate that implements the full integration benchmark. Then each upstream firm has an incentive to lower (raise) its own linear royalty. This increases total industry profit. Hence, by raising the fixed fee of its royalty scheme at the same time, the firm would be better off. Thus, the only symmetric subgame perfect equilibrium in which all downstream firms produce must have identical linear royalties for all upstream firms that sum up to r^{FI}.

(c) Consider an equilibrium candidate where all vertically integrated choose identical linear royalties the sum of which induces the full integration outcome and identical fixed fees the sum of which extracts all profits of the downstream firms/divisions. Let $p^M(k)$ denote the price chosen by a fully integrated monopolist with marginal cost k and let $\Pi^M(k)$ denote his monopoly profit on the downstream market. If the symmetric linear royalties r implement the monopoly price downstream, it must be the case that $(n-1)\cdot r + k < p^M(k)$. Note that each firm makes a profit of $\frac{1}{m}\cdot\Pi^M(k)$ in the candidate equilibrium.

Consider now the following deviation of firm 1. It raises its fixed fee to infinity, so that no other firm can afford to license its patent. Thus, firm 1 becomes a monopolist on the downstream market. It has to pay fixed fees to the other firms that are equal to its downstream profit if it had not deviated. Note that this is bounded above by $\frac{1}{m}\cdot\Pi^M(k)$. On the other hand, it now monopolizes the downstream market, so it will make at least the profit of a monopolist with marginal cost of $(n-1)\cdot r + k < p^M(k)$. These profits are bounded below by the profits of a monopolist with marginal cost $p^M(k)$ (note that $p^M(k)$ is independent of n). Denote the profit of a monopolist on the downstream market with marginal cost $p^M(k)$ as $\Pi^M\left(p^M(k)\right)=\underline{\Pi}>0$.

Hence, a deviation is profitable if $\underline{\Pi}-\frac{1}{m}\Pi^M(\text{k})>\frac{1}{m}\Pi^M(\text{k}) \quad \Leftrightarrow \quad \underline{\Pi}>\frac{2}{m}\Pi^M(\text{k})$. Note that for all k, $\frac{2}{m}\Pi^M(k)$ goes to zero as m goes to infinity. Thus, there exists an $\overline{m}$ such that for all $m \geq \underline{m}$ the deviation is profitable. If $m \geq \underline{m}$ there does not exist a symmetric pure strategy subgame perfect equilibrium that implements the monopoly outcome. *Q.E.D.*

Proof of Proposition 5: By Assumption 2 the quantity sold on the downstream market increases after entry for any given cumulative royalty rate. Thus, if a horizontally integrated upstream firm keeps its royalty rate constant it must benefit from entry. The option to adjust the royalty rate may increase profits even further.

To see that the effect of entry on a vertically integrated firm is ambiguous and that a VI firm may even foreclose an entrant by charging a prohibitively high royalty rate consider the following example:[15] There is one vertically integrated incumbent (firm 1) and one potential entrant on the downstream market (firm 2). Firm 2 requires a license for an essential patent from firm 1. The two firms offer differentiated products and compete in prices. Inverse demand functions are given by $p_1=a-q_1-sq_2$ and $p_2=a-q_2-sq_1$ where p_i is the price charged by firm i, q_i is the quantity sold by firm i and $s\in[0,1]$ is a substitution coefficient. The smaller s the more are the two goods differentiated. Both firms have identical marginal costs that are normalized to 0. There is no sunk cost of entry. The royalty rate charged by firm 1 is denoted by r. Profit functions for the downstream market are given by

$$\Pi_1^D = \quad p_1 q_1 = \quad \frac{p_1\left(a(1-s)-p_1+sp_2\right)}{1-s^2}$$

$$\Pi_2 = \quad (p_2-r)q_2 = \quad \frac{p_2\left(a(1-s)-p_2+sp_1\right)-r(a-p_2+sp_1)}{1-s^2}$$

Solving for the Nash equilibrium in prices yields

$$p_1=\frac{a(2-s-s^2)+rs}{4-s^2} \quad \text{and} \quad p_2=\frac{a(2-s-s^2)+2r}{4-s^2}$$

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[15] A similar example has been used in a different context by Wang and Yang (1999, Setion III).

Thus, when firm 1 decides on its royalty rate it maximizes $\Pi_1 = rq_2 + \Pi_1^D$. Assuming an interior solution the optimal royalty rate is given by

$$r^* = \frac{a(4-2s-s^2)(2-s-s^2)}{8-7s^2+s^4}$$

Thus, the profit of the vertically integrated firm if there is entry is given by

$$\Pi = \frac{a^2(12+4s-11s^2-5s^3)}{4\left(8+8s-7s^2-7s^3+s^4+s^5\right)}$$

Alternatively, the vertically integrated firm could charge a prohibitively high royalty rate that keeps the entrant out of the market. In this case the vertically integrated firm is a monopolist, chooses $p_1 = \frac{1}{2}a$, and makes profit $\Pi^M = \frac{1}{4}a^2$. It is straightforward to show numerically that the vertically integrated firm strictly prefers to foreclose entry if $s > 0.64$, i.e. if the two products are sufficiently close substitutes. *Q.E.D.*

References

Amir, Rabah (1996), "Cournot Oligopoly and the Theory of Supermodular Games," *Games and Economic Behavior*, Vol. 15, 132-148.

Aoki, Reiko and Nagaoka, Sadao (2004), "The Consortium Standard and Patent Pools," Hibotsubashi University Research Unit for Statistical Analysis in Social Sciences, Working Paper No. 32.

Cournot, Auguste A. (1838), *Researches into the mathematical principles of the theory of wealth*, English edition of *Researches sur les principles mathematiques de la theorie des richesses.* New York: Kelley.

Debreu, Gerard (1952), "A Social Equilibrium Existence Theorem," *Proceedings of the National Academy of Sciences*, Vol. 38, 886-893.

Dixit, Avinash (1986), "Comparative Statics for Oligopoly," *International Economic Review*, Vol. 27 (1), 107-122.

Farrell, Joseph and Simcoe, Tim (2012), "Choosing the Rules for Consensus Standardization." *The RAND Journal of Economics*, Vol. 43(2), 235-252.

Ganglmair, Bernhard and Tarantino, Emanuele (2011), "Patent Disclosure in Standard Setting," University of Texas, *mimeo*.

Gilbert, Richard J. (2004), "Antitrust for Patent Pools: A Century of Policy Evolution," *Stanford Technology Law Review*, Vol. 3, available at http://stlr.stanford.edu .

Gilbert, Richard J., and Katz, Michael L. (2011), "Efficient Division of Profits from Complementary Innovations", *International Journal of Industrial Organization*, Vol. 29(4), 443-454.

Höffler, Felix and Kranz, Sebastian (2011), "Imperfect Legal Unbundling of Monopolistic Bottlenecks", *Journal of Regulatory Economics*, Vol. 39 (3), 273-292.

Kim, Sung-Hwan (2004): "Vertical Structure and Patent Pools", *Review of Industrial Organization* 25, 231-250.

Layne-Farrar, Anne and Lerner, Josh (2011), "To Join or not to Join: Examining Patent Pool Participation and Rent Sharing Rules," *International Journal of Industrial Organization*, Vol. 29(2).

Layne-Farrar, Anne, Padilla, Jorge, and Schmalensee, Dick (2007), "Pricing Patents for Licensing in Standard Setting Organisations: Making Sense of FRAND Commitments," *Antitrust Law Journal*, Vol 75.

Layne-Farrar, Anne and Klaus M Schmidt. 2011. "Licensing Complementary Patents: "Patent Trolls," Market Structure, and Excessive Royalties." *Berkeley Technology Law Review*, Vol. 25, 1123–1145.

Lemley, Mark, and Shapiro, Carl (2007), "Patent Holdup and Royalty Stacking," *Texas Law Review*, available at http://faculty.haas.berkeley.edu/shapiro/stacking.pdf .

Lerner, Josh and Tirole, Jean (2004), "Efficient Patent Pools," *American Economic Review*, Vol. 94(3), 691-711.

Lerner, Josh, Strojwas, Marcin, and Tirole, Jean (2006), "The Design of Patent Pools: The Determinants of Licensing Rules," *RAND Journal of Economics*.

Novshek, William (1985): "On the Existence of Cournot Equilibrium", *Review of Economic Studies* 52, 85-08.

Ordover, Janusz A., Saloner, Garth, and Salop, Steven C (1990): "Equilibrium Vertical Foreclosure", *American Economic Review* 80, 127-141.

Reisinger, Markus and Tarantino, Emanuele (2010), "Vertical Integration with Complementary Inputs," University of Bologna, *mimeo*.

Rey, Patrick and David Salant (2012), "Abuse of Dominance and Licensing of Intellectual Property," *International Journal of Industrial Organization*, Vol. 30, 518-27.

Salop, Steven C. and Scheffman, David T. (1983): "Raising Rivals' Costs", *American Economic Review, Papers and Proceedings* 73, 267-271.

Salop, Steven C. and Scheffman, David T. (1987): "Cost-Raising Strategies", *Journal of Industrial Economics* 36, 19-34.

Shapiro, Carl (2001): "Navigating the Patent Thicket: Cross Licenses, Patent Pools, and Standard Setting,", in: Jaffe, Adam. Lerner, Joshua, and Stern, Scott (eds.) *Innovation Policy and the Economy*, Vol. 1, Cambridge, Mass.: MIT-Press.

Simcoe, Timothy (2012), "Standard Setting Committees: Consensus Governance for Shared Technology Platforms," *American Economic Review*, 102(1): 305–36.

Swanson, Daniel G.and Baumol, William J. (2005), "Reasonable and Nondiscriminatory (Rand), Royalties, Standards Selection, and Control of market Power," *Antitrust Law Journal*, Vol. 73.

Tarantino, Emanuele (2009), "Inefficient Exclusion in Standard Setting Organizations," University of Bologna, *mimeo*.

Wang, X. Henry and Yang, Bill Z. (1999), "On Licensing under Bertrand Competition," *Australian Economic Papers*, Vol. 38, 106-119.